Enhanced Bayesian Network for Reliability Assessment: Application to Salt Domes as Disposal Sites for Radioactive Waste Problem

Andrea Perin

Institute for Risk and Reliability
Leibniz Universität Hannover

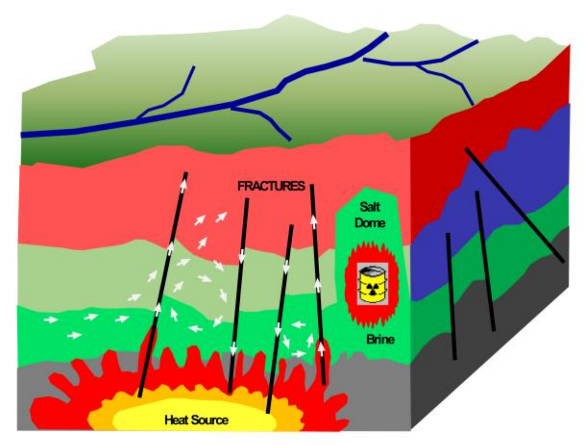
Problem Introduction

Risk Assessment of salt domes as long-term radioactive waste disposal

Project

Project goal is to develop a platform for the probabilistic assessment of unintentional leakage of radioactive materials associated with deep repositories:

- Enhanced Bayesian Network (EBN) for risk assessment
- Thermal-Hydraulic-Components (THC) model for density-driven flow



Salt Dome Problem

- Radionuclide transport inside a salt dome is **Thermal-Hydraulic-Components (THC)** transport phenomenon:
  - **Thermal**:
    Geothermal gradient + Heat generated by waste decay
  - **Hydraulic**:
    Groundwater flow in fractured porous media
  - **Components**:
    Salt and radionuclides are the 2 components of interest

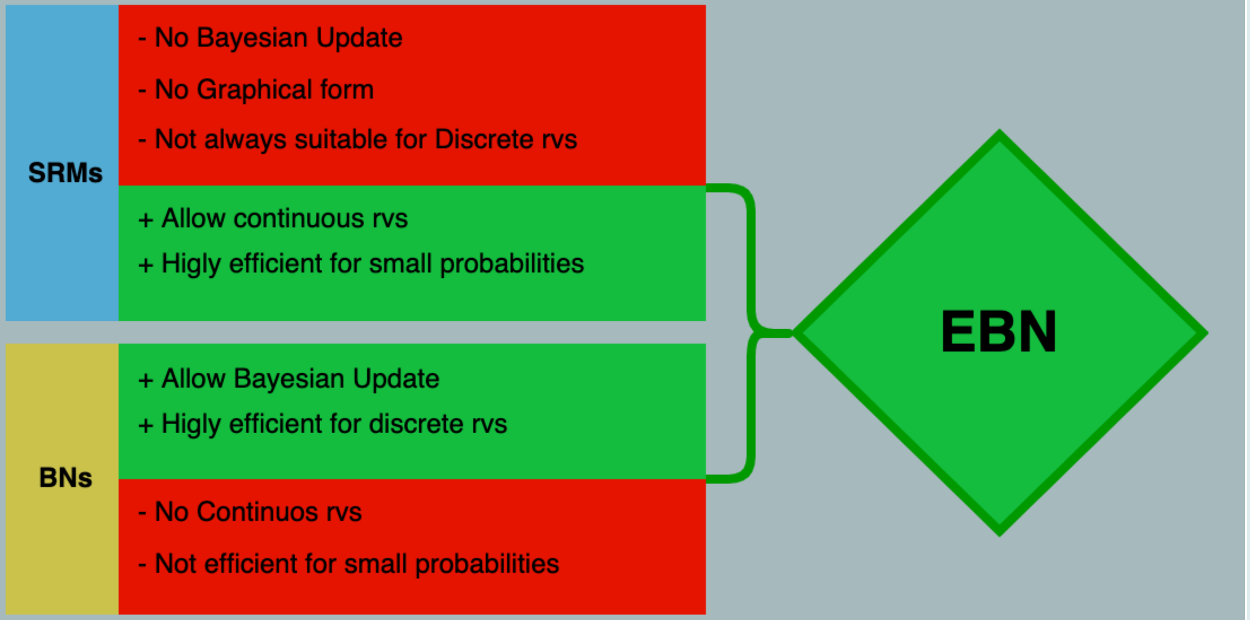
Salt → salt dissolution affects flow velocity and vice-versa (*Variable density and viscosity flow*)
Radionuclide → Quantity of interest for safe/fail state definition

- Reuired safety constrain: uncontaminate biosphere over a time span of 1M years

Risk Assessment through Enhanced Bayesian Network (EBN)

Risk Assessment (EBN) – General Concepts

IDEA



SRMs

- No Bayesian Update
- No Graphical form
- Not always suitable for Discrete rvs

+ Allow continuous rvs
+ Higly efficient for small probabilities

BNs

+ Allow Bayesian Update
+ Higly efficient for discrete rvs

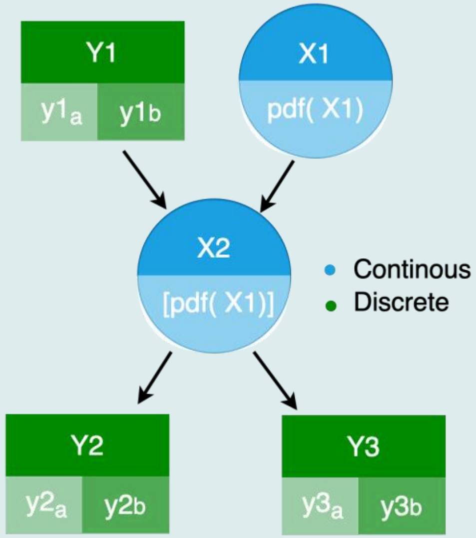
- No Continuos rvs
- Not efficient for small probabilities

EBN

eBNs (*BNs Enhanced with SRM*) are a tool able to:

- Implement Discrete and Continuous rvs
- With arbitrary distributions
- And any dependency

HOW



● Continous
● Discrete

Formally

- **Discrete nodes** have a *finite sample space*
- **Continuous nodes** are *vectors of continuous rvs*
- **System pdf** is expressed by the combined effect of continous and discrete rvs

System pdf:

$$f(\boldsymbol{Z}) = \prod_{Yi \in \boldsymbol{Y}} f(yi|pa[Yi]) + \prod_{Xi \in \boldsymbol{X}} f(xi|pa[Xi])$$

The problem of the evaluation of discrete probabilities (or pdf) of each node with at least one continuous parent has the same mathematical form of a Structural Reliability Problem!

Risk Assessment (EBN) – Main Features

Main Structures

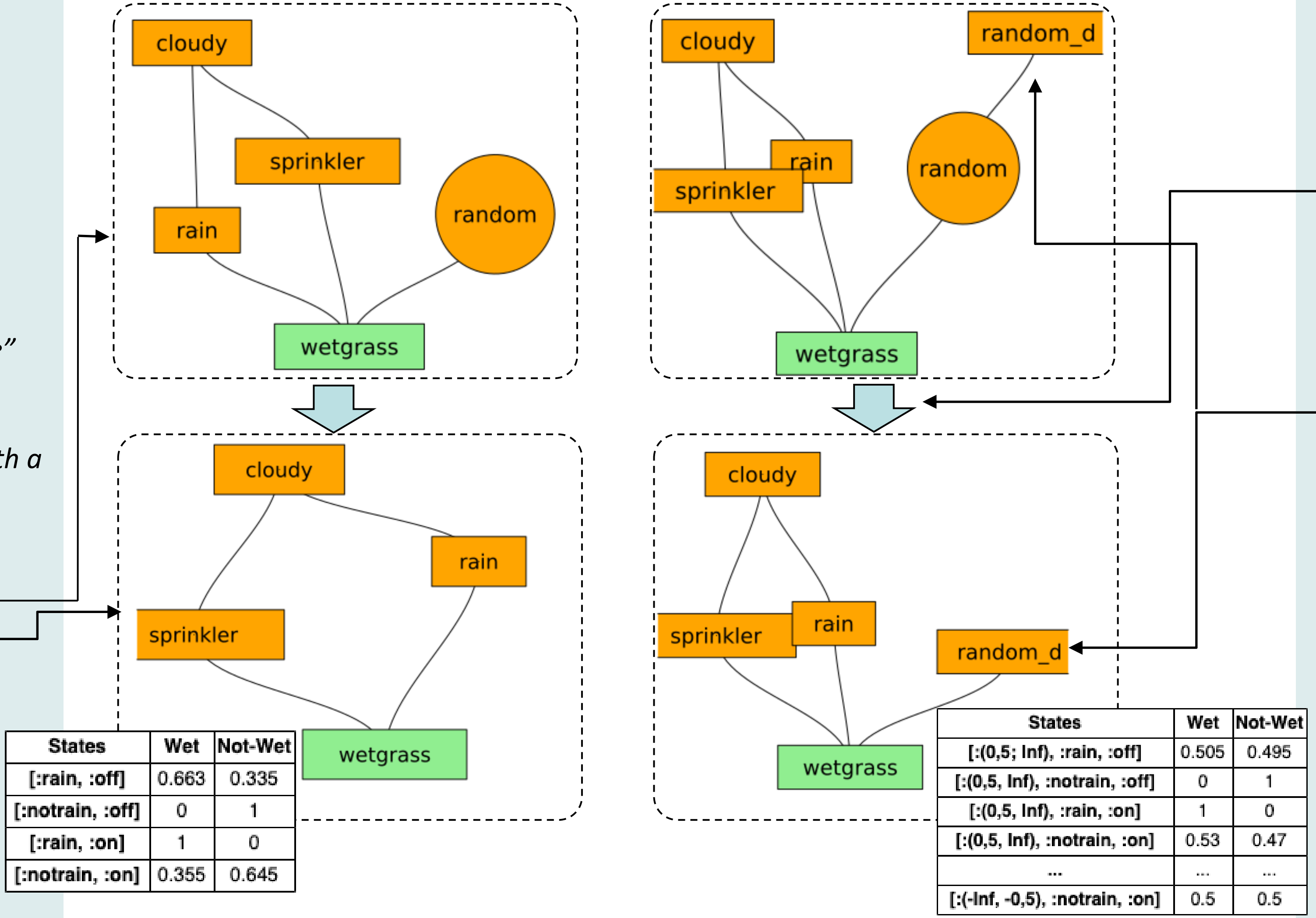
Nodes

- RootNode
- StandardNode
- FunctionalNode

Each node can be ”Discrete” with a discrete Conditional Probability Distribution (dCPD) or “Continuous” with a continuos CPD (cCPD)

Networks

- EnhancedBN
- ReducedBN (rBN)
- BN



States	Wet	Not-Wet
[:rain, :off]	0.663	0.335
[:notrain, :off]	0	1
[:rain, :on]	1	0
[:notrain, :on]	0.355	0.645

States	Wet	Not-Wet
[:(0,5; Inf), :rain, :off]	0.505	0.495
[:(0,5, Inf), :notrain, :off]	0	1
[:(0,5, Inf), :rain, :on]	1	0
[:(0,5, Inf), :notrain, :on]	0.53	0.47
...	...	...
[:(-Inf, -0,5), :notrain, :on]	0.5	0.5

Main Algorithms

Nodes Elimination Algorithm

- Allow the evaluation of the associated rBN
- Remove continuous nodes
- Transfer info to the rBN’s SRPs

Evidence Algorithm

- Discretize continuous node for which future evidence can be available
- Allow Inference over discretized continuous nodes

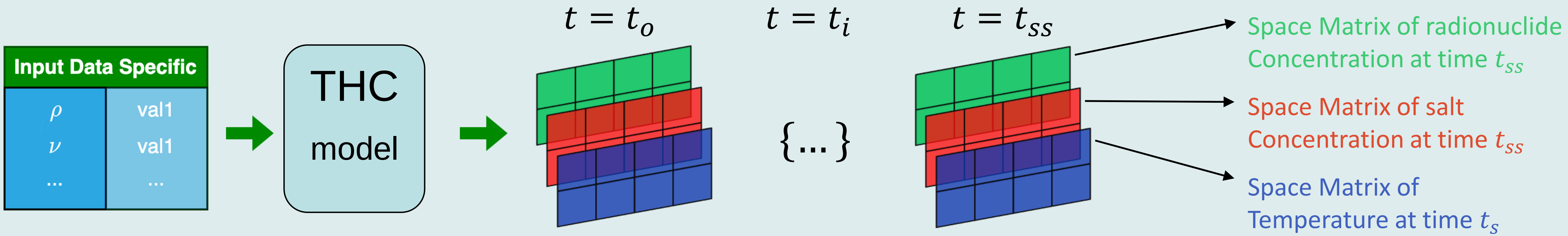
Inference Algorithm

- Allow inference over each node of the resulting BN
- Inference is performed through ”*variable elimination*”
- Computational time is optimized through “*minimal increase in complexity algorithm*”

Implementation

Implementation – THC solver

Model's output



Input Data Specific

ρ	val1
ν	val1
…	…

THC model

$t = t_o$

$t = t_i$

$t = t_{ss}$

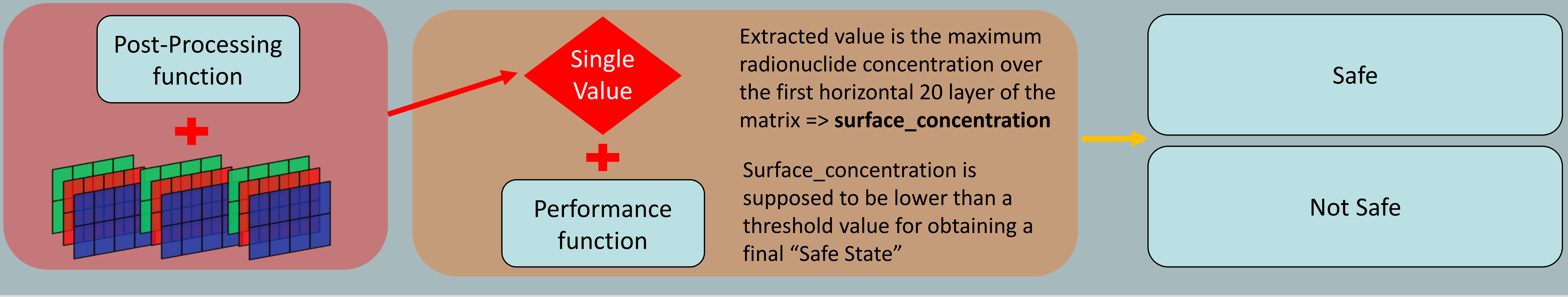
$\{\ldots\}$

Space Matrix of radionuclide Concentration at time t_{ss}

Space Matrix of salt Concentration at time t_{ss}

Space Matrix of Temperature at time t_s

Post-Processing + Performance Function



Post-Processing function

Single Value

Performance function

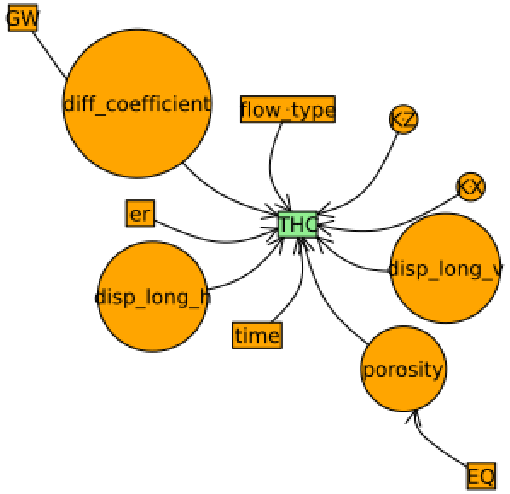
Extracted value is the maximum radionuclide concentration over the first horizontal 20 layer of the matrix => **surface_concentration**

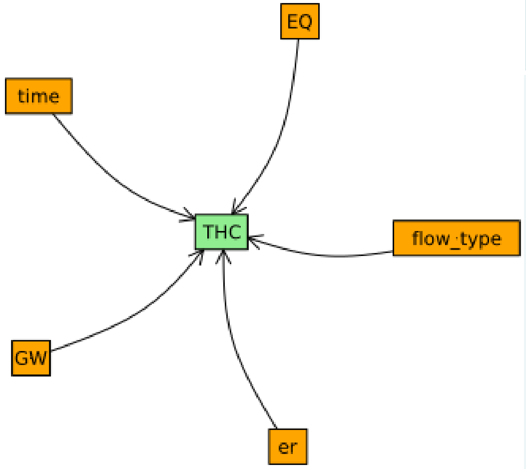
Surface_concentration is supposed to be lower than a threshold value for obtaining a final "Safe State"

Safe

Not Safe

Implementation – EBN





Earthquake (EQ):

- DiscreteRootNode
  - :happen= $10e^{-5}$
  - :not_happen = $1 - 10e^{-5}$

Porosity (porosity)

- Child of Earthquake
- ContinuousStandardNode
  - :happen => trunc N$(3; 0.5)$
  - :not_happen => trunc N$(1; 0.5)$

Global-warming (GW)

- DiscreteRootNode
  - :warming => 0.7
  - :astoday => 0.2
  - :cooling => 0.1

Diffusion (diff_coefficient)

- Child of Global-Warming
- ContinuousStandardNode
  - :warming => trunc N$(2e^{-6}; e^{-6})$
  - :astoday => trunc N$(2e^{-8}; e^{-7})$
  - :cooling => trunc N $(2e^{-9}; e^{-6})$

Extreme Rain (er)

- DiscreteRootNode
  - :extremerain => 0.4
  - :no_extremerain => 0.6
- Influenced parameters
  - :extremerain => :head = 1.2
  - :no_extremerain => :head = 0.8

Hydraulic Conductivity x-direction (KX)

- ContinuousRootNode(truncated N$(9.81e^{-6}; e^{-4})$)

Hydraulic Conductivity z-direction (KZ)

- ContinuousRootNode(truncated N$(9.81e^{-6}; e^{-4})$)

Time Scenario

- Short (10^5 days) or Long (10^7 days)

Longitudinal Dispersivity vertical(disp_long_v)

- ContinuousRootNode(Uniform$(10; 60)$)

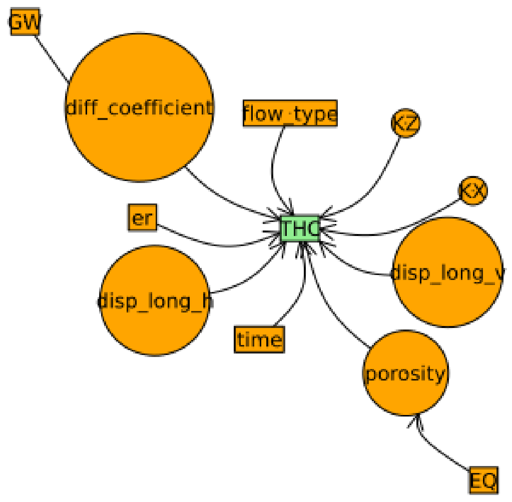
Longitudinal Dispersivity horizontal (disp_long_h)

- ContinuousRootNode(Uniform$(1; 6)$)

Flow Type

- Steady-State or Transient Solution

Implementation – EBN



THC Model's Node

Model:

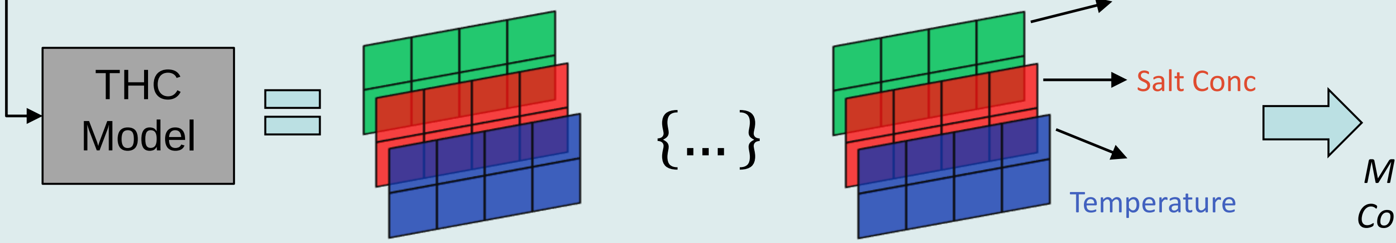
- FE model (60x30)
- No heat source
- Salt and Radionuclide Transport
- Density driven flow simulation

Continuous Parents

- Diffusion Coefficient
- Hydraulic Conductivity X
- Hydraulic Conductivity Z
- Dispersivity Longitudinal Vertical
- Dispersivity Longitudinal Horizontal
- Porosity

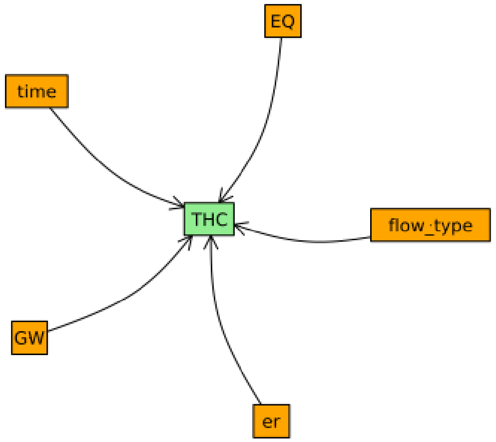
Discrete Parents

- Extreme Rain
- Time Scenario
- Flow type



Maximum Surface Concentration over simulation's time

Implementation – Results



rBN – THC model's Node

- Extreme Rain => 2 states
- Time Scenario => 2 states
- Flow type => 2 states
- Earthquake => 2 states
- Global Warming => 3 states

$2^4 * 3 = 48$ SRPs (200 simulations each)

Performance => Surface Concentration > 0

state	fail	safe
comb 1	0.995	0.005
comb 2	1	0
comb 3	0.87	0.13
comb 4	0.885	0.114
comb 5	0.92	0.079
comb 6	0.875	0.125
comb 7	1	0
comb 8	0.83	0.17
comb 9	0.985	0.015
comb 10	0.99	0.01
comb 11	0.885	0.114
comb 12	0.92	0.079

{ … }

state	fail	safe
comb 22	0.985	0.015
comb 23	0.94	0.06
comb 24	0.98	0.02
comb 25	1	0
comb 26	1	0
comb 27	0.985	0.015
comb 28	0.82	0.18
comb 29	0.93	0.069
comb 30	0.835	0.165
comb 31	0.905	0.094
comb 32	0.895	0.104
comb 33	0.905	0.094

Implementation – Results

rBN – Inference

Time Scenario
$$\begin{cases} p(time\_long|THC\_fail) = 0.57 \\ p(time\_short|THC\_fail) = 0.43 \end{cases}$$

Earthquake
$$\begin{cases} p(earthquake|THC\_fail) = 10^{-5} \\ p(no\_earthquake|THC\_fail) = 0.99999 \end{cases}$$

Global Warming
$$\begin{cases} p(cooling|THC\_fail) = 0.11 \\ p(astoday|THC\_fail) = 0.19 \\ p(warming|THC\_fail) = 0.70 \end{cases}$$

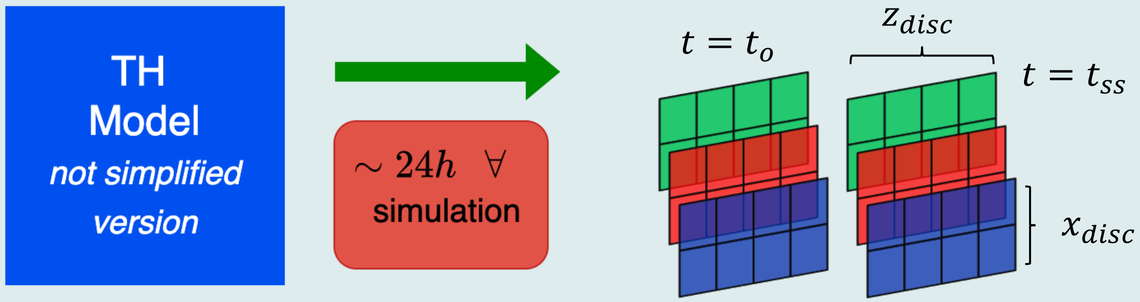
Extreme Rain
$$\begin{cases} p(er|THC\_fail) = 0.54 \\ p(no\_er|THC\_fail) = 0.46 \end{cases}$$

- Deal with multidisciplinary and continuous node
- Allow ”What-if” analysis through Inference algorithm

- Allow “model Update” through Bayesian update.
- Computational cost depends on model only

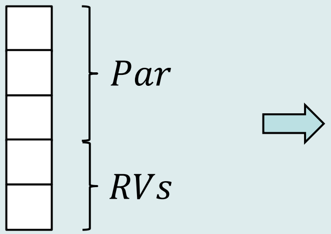
Analysis speed-up through Surrogate Model

FE Models are too computational expensive in a framework where are required to be run several times in different scenarios, especially when low probability of failure have to be established



$Dim = Par + RVs$

$Dim = t_{disc} * x_{disc} * z_{disc} * outputs$

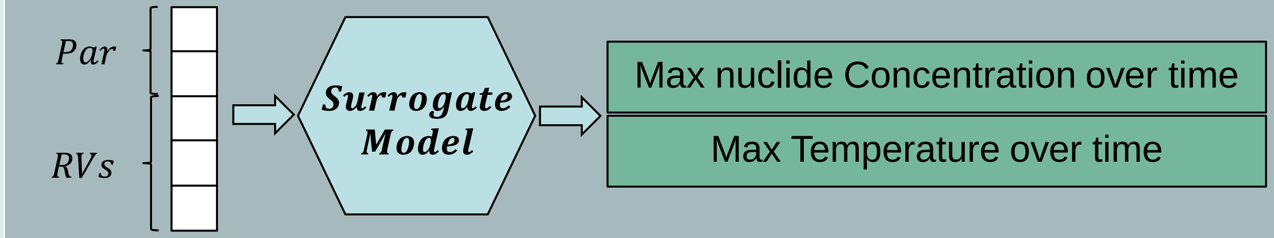


Output	
t_{disc}	1 or 2
x_{disc}	10^2
z_{disc}	10^2
outputs	3 [c_s;T;c_r]

With a 24h simulation we obtain 1 output sample of 10^4 dimension!

Surrogate Output

Reduced Output Dimensionality



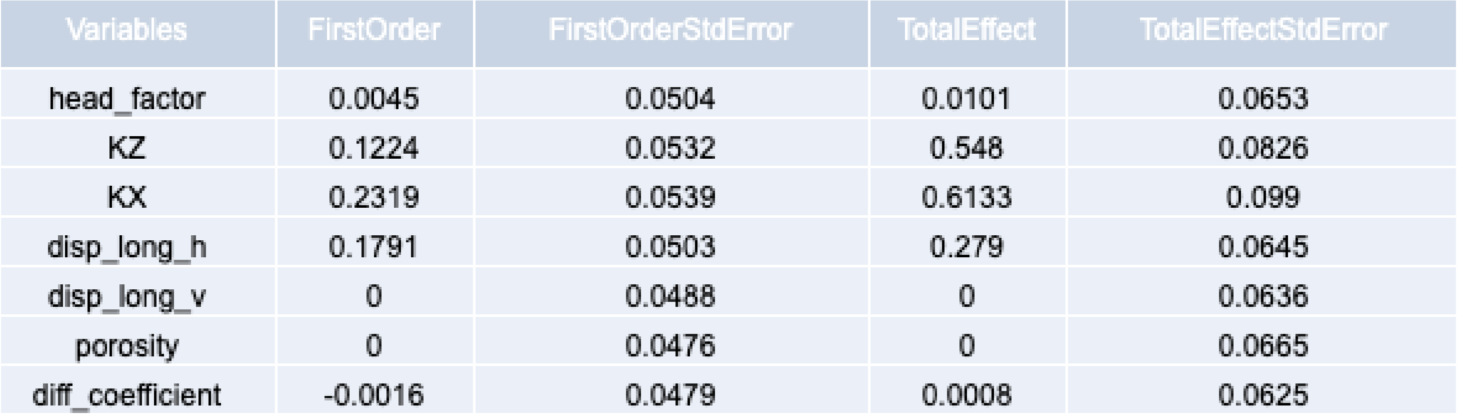
Instead of predicting matrices, **the Surrogate Model will predict a specific value for Radionuclide Concentration and a specific value for the Temperature**

Analysis speed-up through Surrogate Model

Interval Predictor Model (IPM): **No feasible solution for the optimization problem**

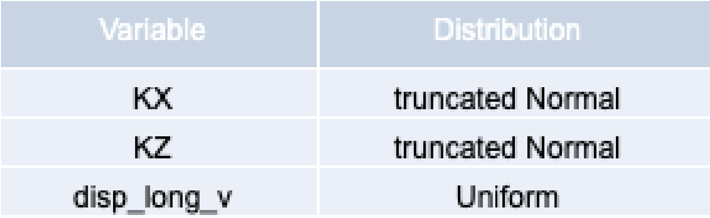
Polynomial Chaos Expansion (PCE) with Iso-probabilistic transformation

Sensitivity Analysis



Variables	FirstOrder	FirstOrderStdError	TotalEffect	TotalEffectStdError
head_factor	0.0045	0.0504	0.0101	0.0653
KZ	0.1224	0.0532	0.548	0.0826
KX	0.2319	0.0539	0.6133	0.099
disp_long_h	0.1791	0.0503	0.279	0.0645
disp_long_v	0	0.0488	0	0.0636
porosity	0	0.0476	0	0.0665
diff_coefficient	-0.0016	0.0479	0.0008	0.0625

PCE Random Variables

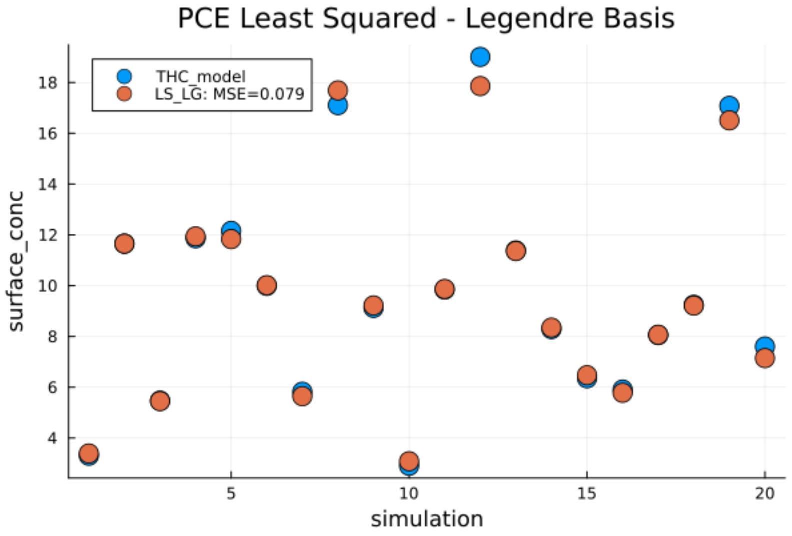


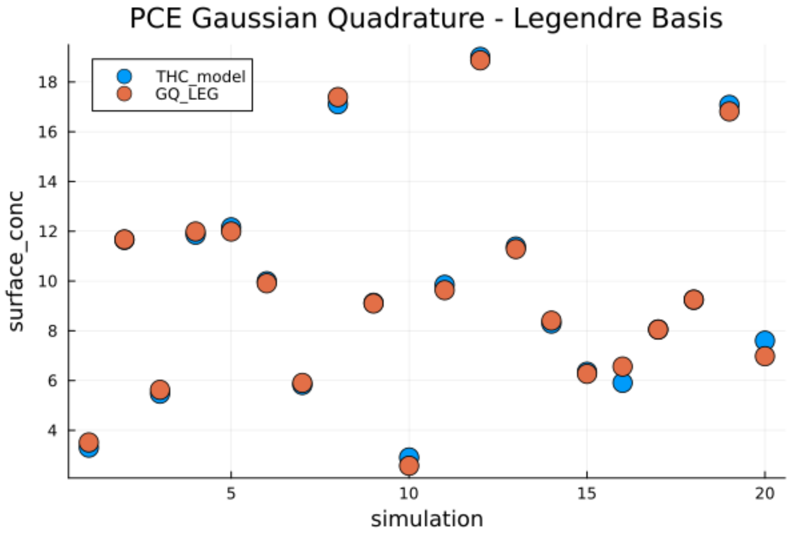
Variable	Distribution
KX	truncated Normal
KZ	truncated Normal
disp_long_v	Uniform

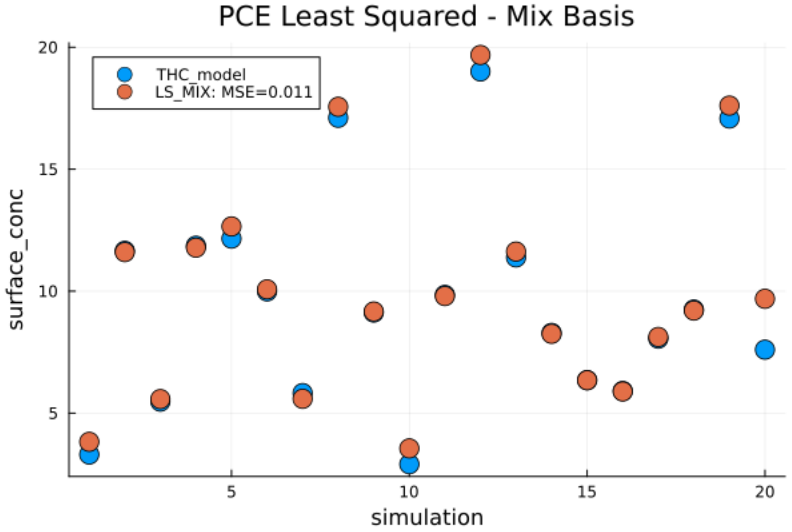
PCEs have been tested with 4 different combination of basis/point determination strategies:

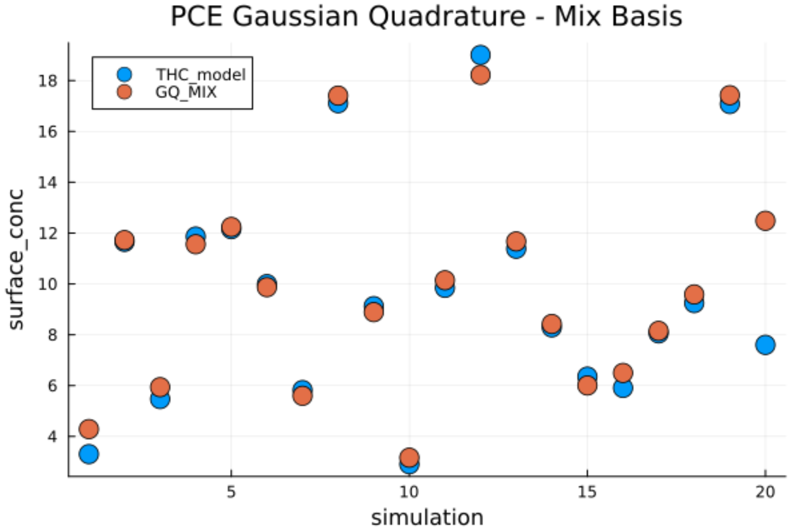
- **Least Square – Legendre Basis Only**
- **Least Square – Mixed Legendre/Hermite Basis**
- **Gaussian Quadrature – Legendre Basis Only**
- **Gaussian Quadrature – Mixed Legendre/Hermite Basis**

Analysis speed-up through Surrogate Model


PCE Least Squared - Legendre Basis


PCE Gaussian Quadrature - Legendre Basis


PCE Least Squared - Mix Basis


PCE Gaussian Quadrature - Mix Basis

Test MSE with 150 random samples

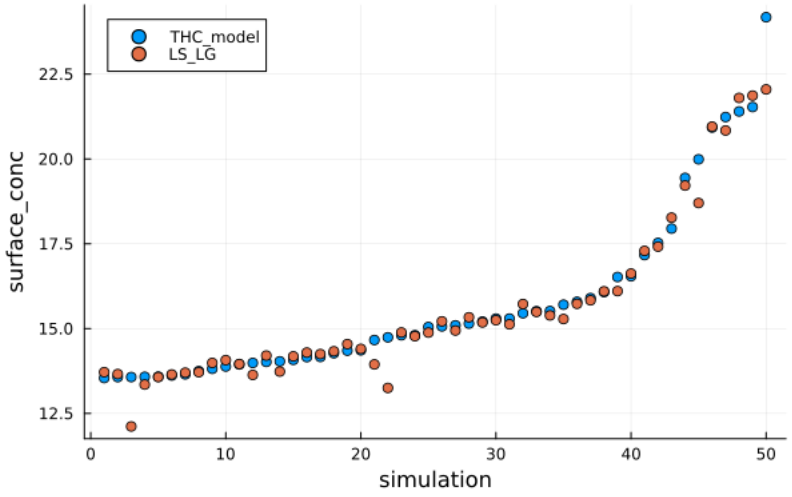
LS_leg	LS_mix	GQ_leg	GQ_mix
0.339	0.268	0.269	0.352

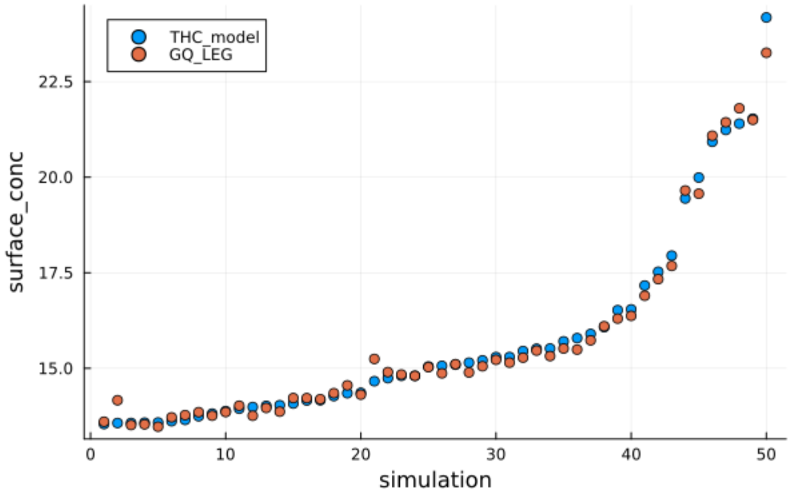
All the the combinations of basis and points determination algorithm are obtained with:

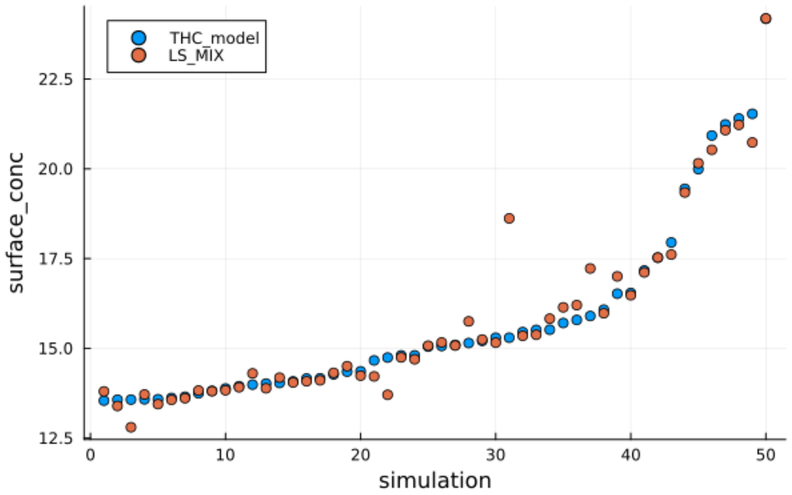
- degree 6
- 216 samples (equal to ones required by Gaussian Quadrature)

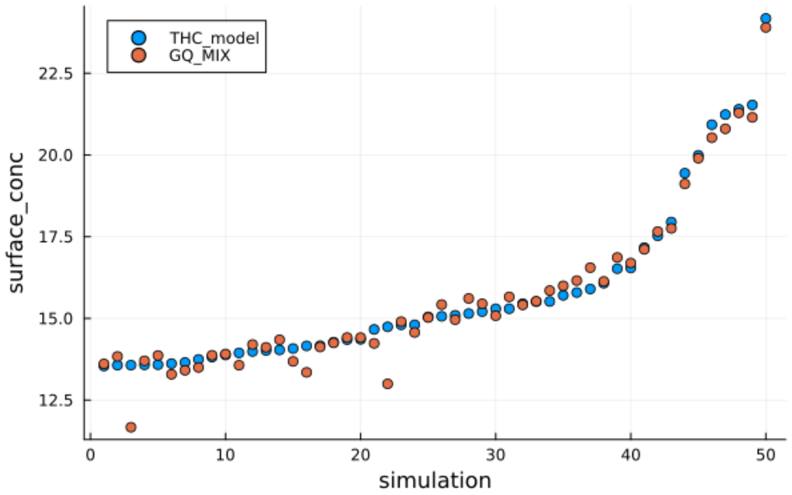
All the the combinations of basis and points determination algorithm seems to perform good with a slightly better MSE for Least Squared and Mix Basis (Legendre for Uniform distributions and Hermite for Gaussian distributions)

Analysis speed-up through Surrogate Model


PCE Least Squared - Legendre Basis


PCE Gaussian Quadrature - Legendre Basis


PCE Least Squared - Mix Basis


PCE Gaussian Quadrature - Mix Basis

Test MSE 100 samples in failure region

LS_leg	LS_mix	GQ_leg	GQ_mix
0.255	0.342	0.057	0.213

All the the combinations of basis and points determination algorithm are obtained with:

- degree 6
- 216 samples (equal to ones required by Gaussian Quadrature)

PCE with Gaussian Quadrature and degree 6 (216 samples) is the one that perform better in prediction of system failure state.

Conclusions & Outlooks

Conclusions & Outlooks

RADON Project

- In-depth analysis of the events (eBN nodes) and their influences on THC model's inputs:
    - *NEA report - Updating the NEA International FEP List An Integration Group for the Safety Case (IGSC) Technical Note*
    - *Projekt ANSICHT FEP Katalog für das Endlagerstandortmodell SÜD*

EBN

- *Interval Probability for Discrete Nodes*
  => CredalNetwork
- *P-Boxes for Continuous Nodes*
- *Gaussian Process to model the error of PCE*
  => Interval model

ACKNOWLEDGMENTS

Funded by *Bundesgesellschaft für Endlagerung* (**BGE**)
the federal company for radioactive waste disposal