Enhanced Bayesian Network for Reliability Assessment with Interval Probability

Application to Salt Domes as Disposal Sites for Radioactive Waste

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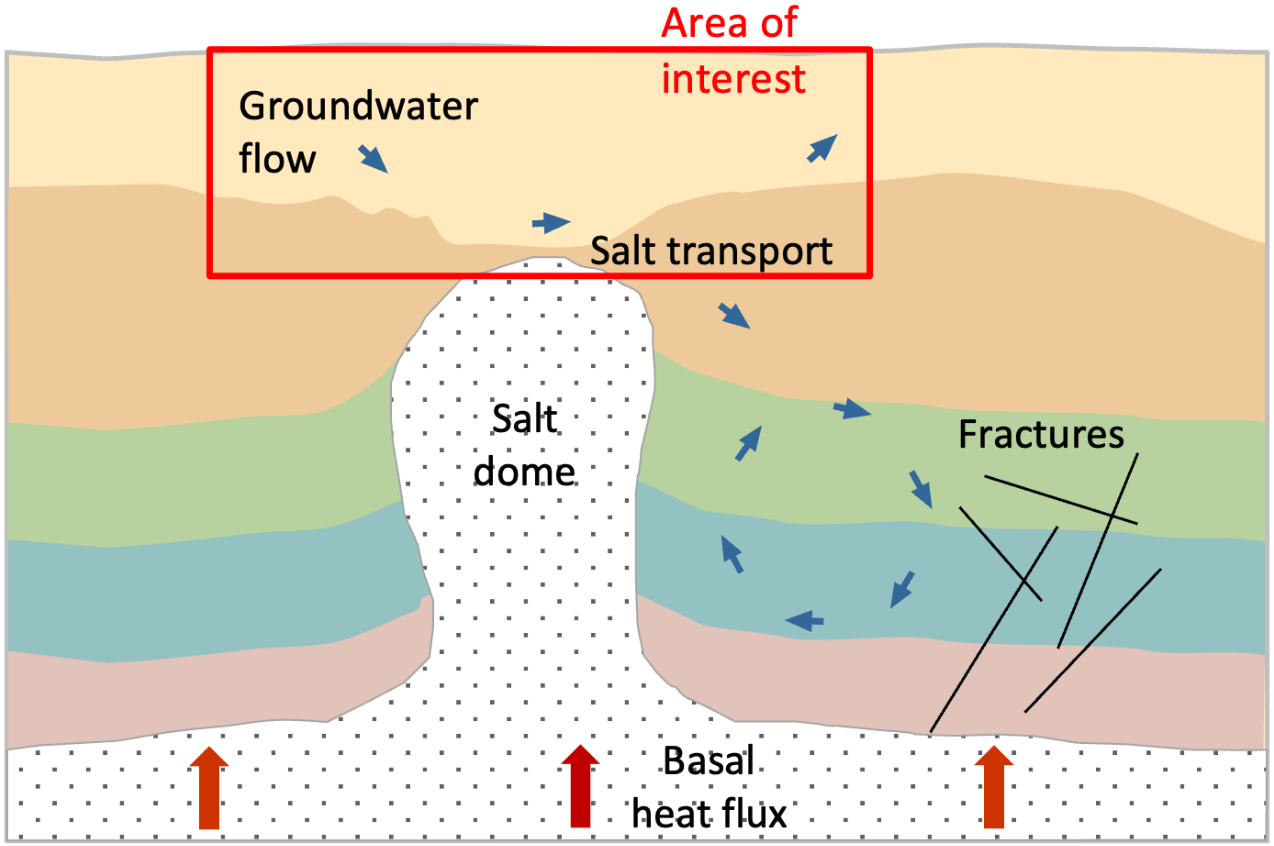
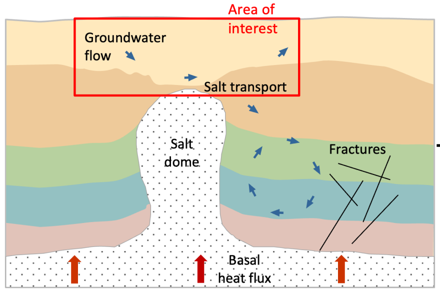
Starting Point



Risk Assessment of **complex structutures** subjected to **uncertainties**

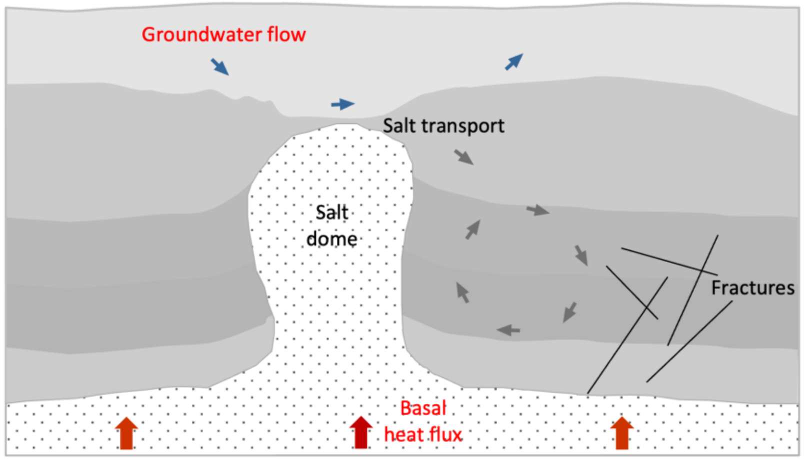
Finite Element (FE) model: radionuclide migration due to a *density-driven groundwater flow*

Risk Assessment

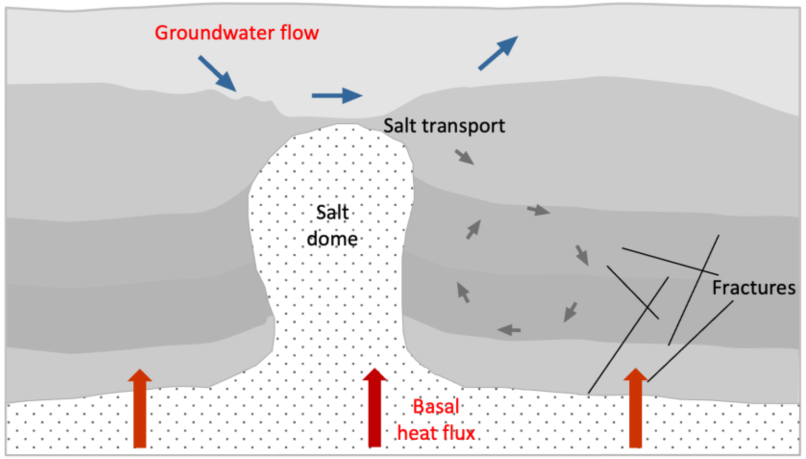


Different Scenarios

A



B

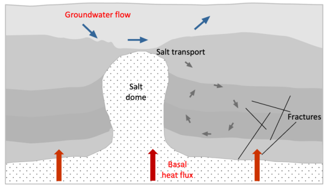


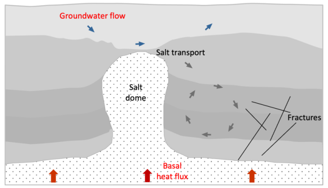
Determinist precisely known parameters may have different values in different scenarios

e.g. $BasalHeatFlux_A < BasalHeatFlux_B$

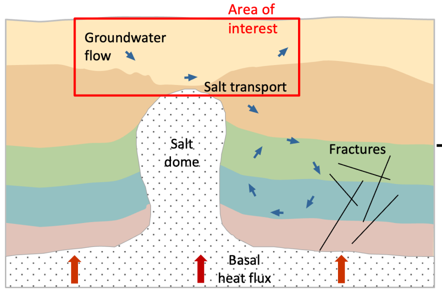
Risk Assessment

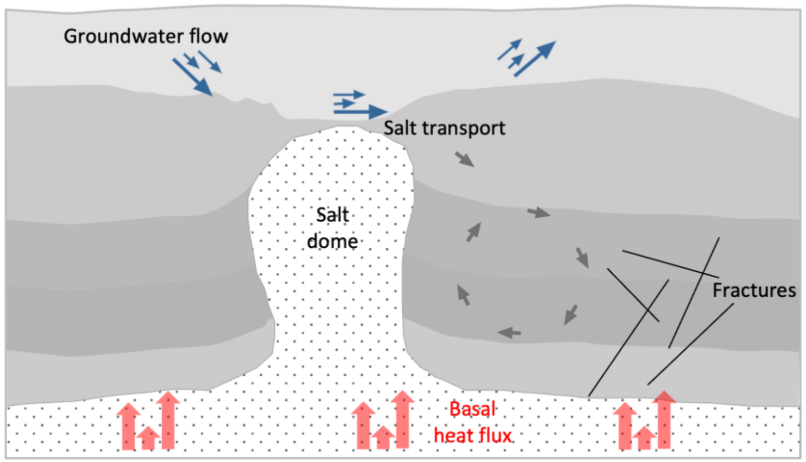
Different Scenarios





Uncertainties

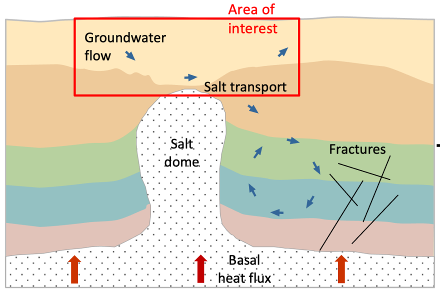




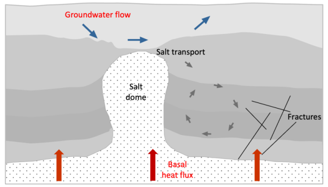
Some quantities may be not precisely know
e.g. “*Basal Heat-Flux*” is a random variable

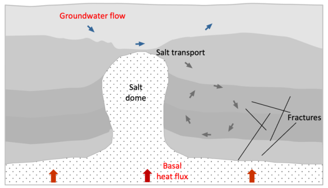
this lack of knowledge must be addressed through uncertainty quantification methodology

Risk Assessment

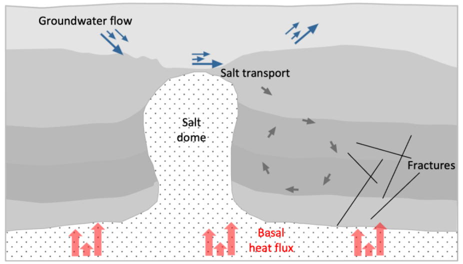


Different Scenarios





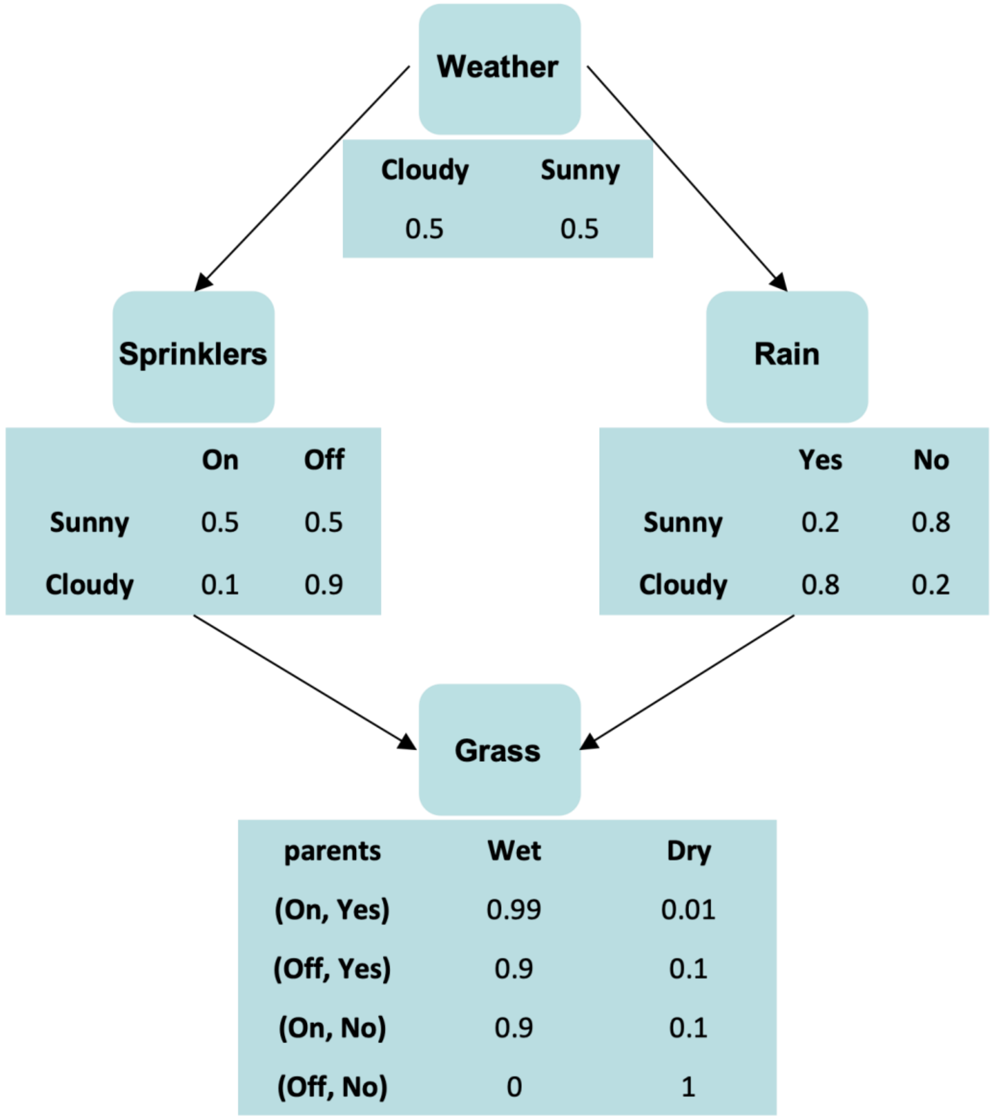
Uncertainties



Tool for dealing with uncertainties in different scenarios:

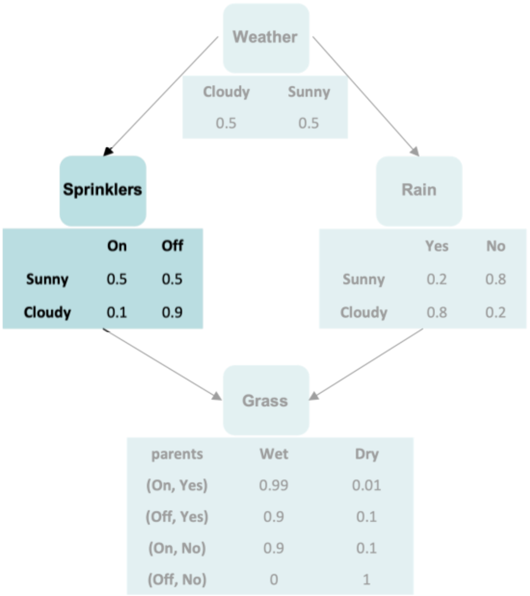
Enhanced Bayesian Network!

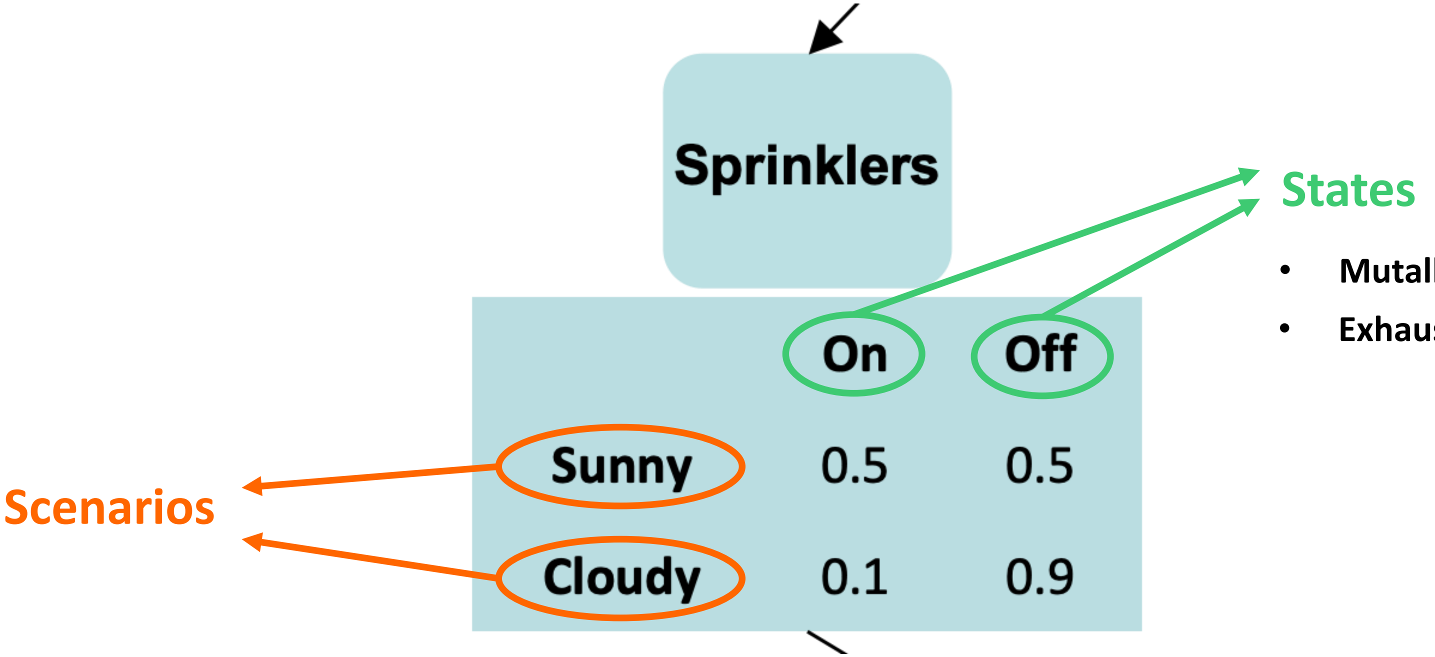
Introduction – BN



- Direct Acyclic Graph (DAG)
- “*Node*”= Each element of the network
- “*Root Node*” = Nodes without parents
- Arrow represent functional relationship
- “*Functional Nodes*” = Nodes defined through a **model** or a **function**
- Each node is defined through a Conditional Probability Table (CPT)

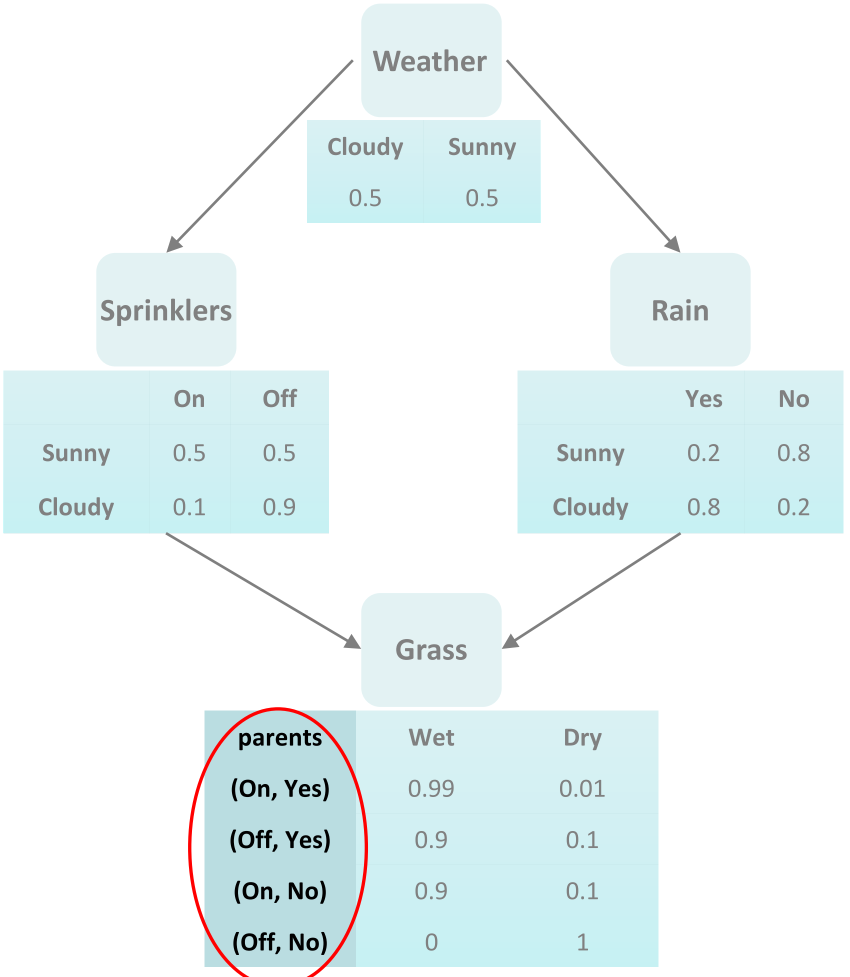
Introduction – BN





- Mutally exlusive
- Exhaustives

Introduction – BN



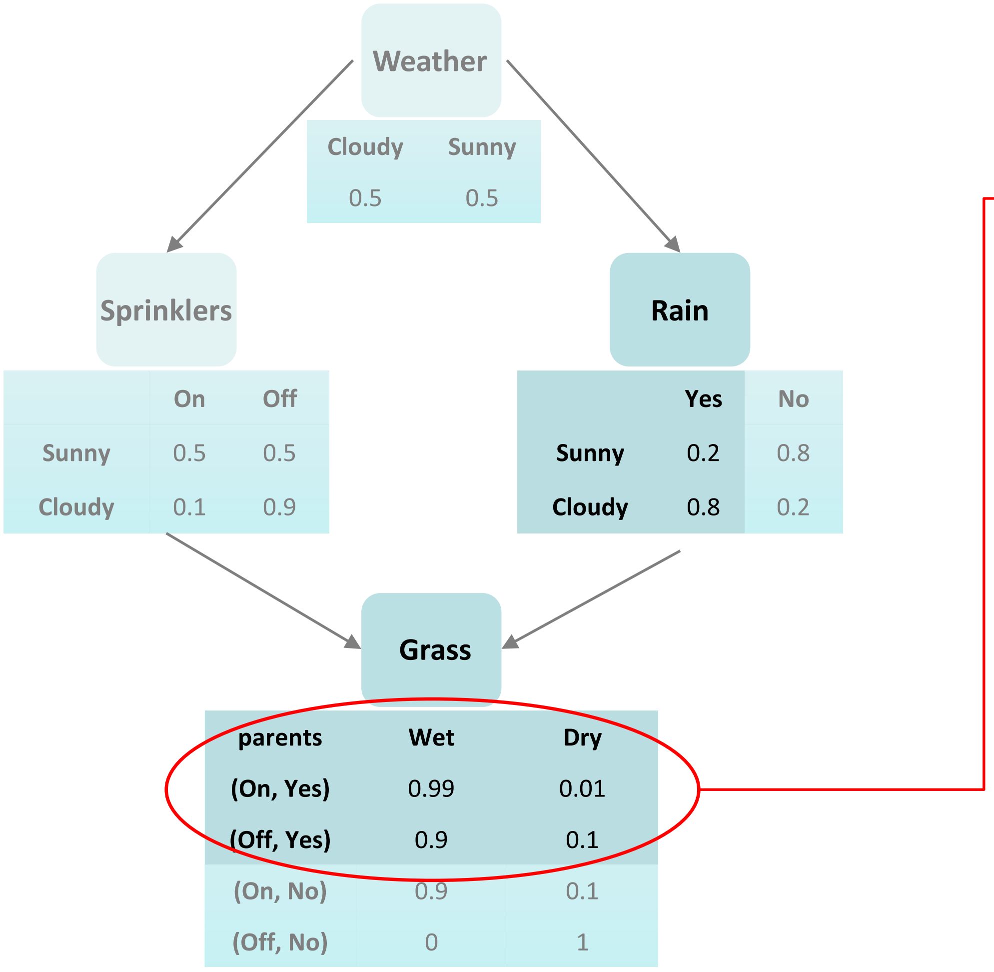
Cloudy	Sunny
0.5	0.5

	On	Off
Sunny	0.5	0.5
Cloudy	0.1	0.9

	Yes	No
Sunny	0.2	0.8
Cloudy	0.8	0.2

parents	Wet	Dry
(On, Yes)	0.99	0.01
(Off, Yes)	0.9	0.1
(On, No)	0.9	0.1
(Off, No)	0	1

✓ **Different Scenario Analysis**



Cloudy	Sunny
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(On, No)	0.9	0.1
(Off, No)	0	1

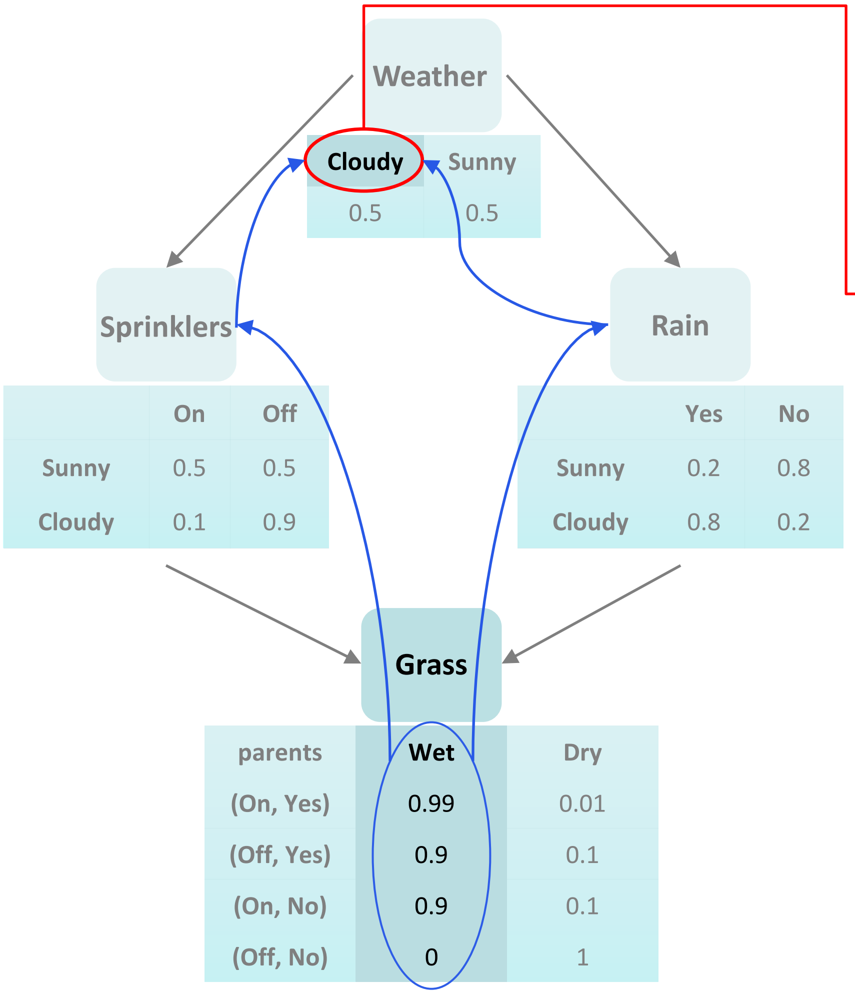
- ✓ Different Scenario Analysis
- ✓ **Evidence Analysis**

Allows probability **update, given** one or more specific node state

e.g. answers the question *“What is the probability of having wet grass, knowing that it had rained?”*

$$P(Wet \mid Rain = Yes)$$

Introduction – BN



	On	Off
Sunny	0.5	0.5
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	Yes	No
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(On, No)	0.9	0.1
(Off, No)	0	1

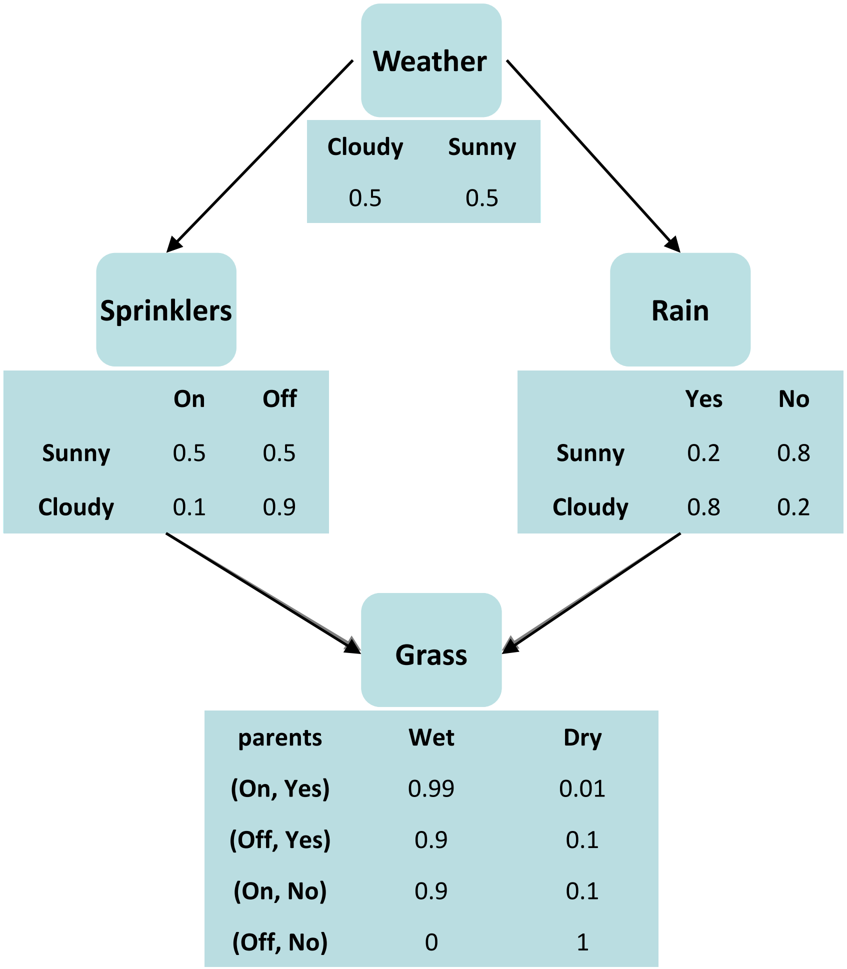
- Different Scenario Analysis
- Evidence Analysis
- **Inference Analysis**

Allows **backward** analysis of the network, i.e. computes the probability, of *any node, to be in any specific state, knowing that a failure took place*

e.g. answers the question *"What is the probability of the weather node in the 'Cloudy' states, knowing that the grass is wet?"*

$$P(Weather = Sunny \mid WetGrass)$$

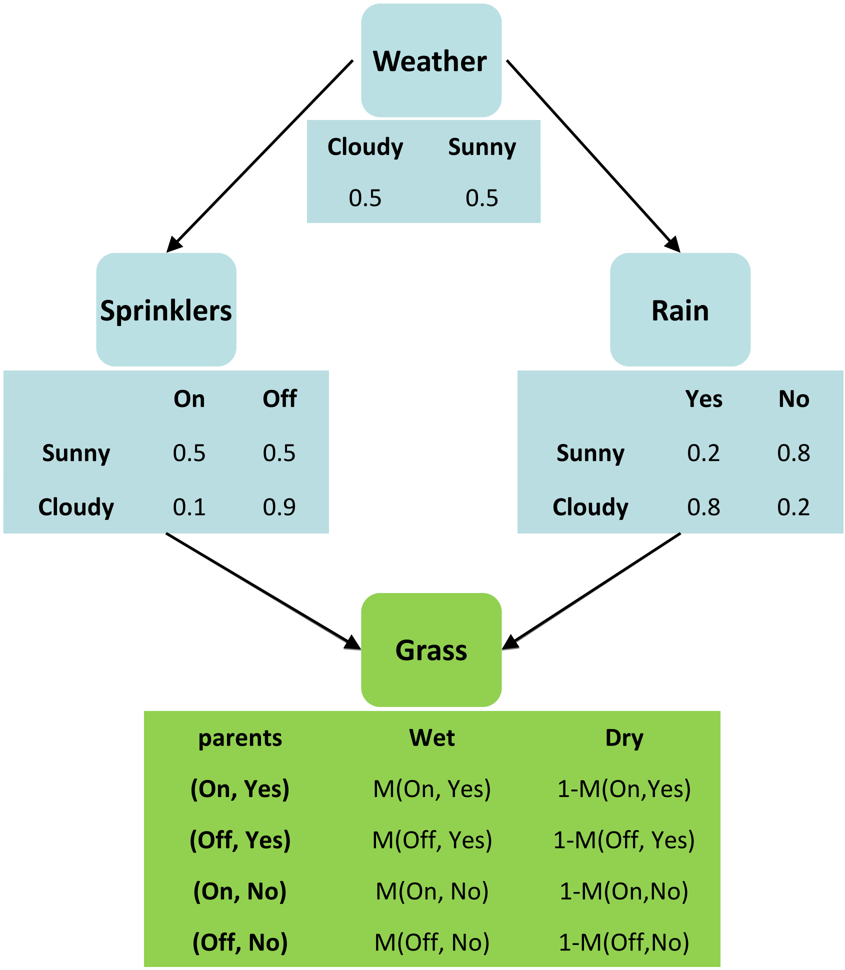
Introduction – BN



- ✓ Different Scenario Analysis
- ✓ Evidence Analysis
- ✓ Inference Analysis

- ✗ **No Continuous Event = No Uncertainties**

Introduction – BN



Cloudy	Sunny
0.5	0.5

	On	Off
Sunny	0.5	0.5
Cloudy	0.1	0.9

	Yes	No
Sunny	0.2	0.8
Cloudy	0.8	0.2

parents	Wet	Dry
(On, Yes)	M(On, Yes)	1-M(On,Yes)
(Off, Yes)	M(Off, Yes)	1-M(Off, Yes)
(On, No)	M(On, No)	1-M(On,No)
(Off, No)	M(Off, No)	1-M(Off,No)

✓ Different Scenario Analysis

✓ Evidence Analysis

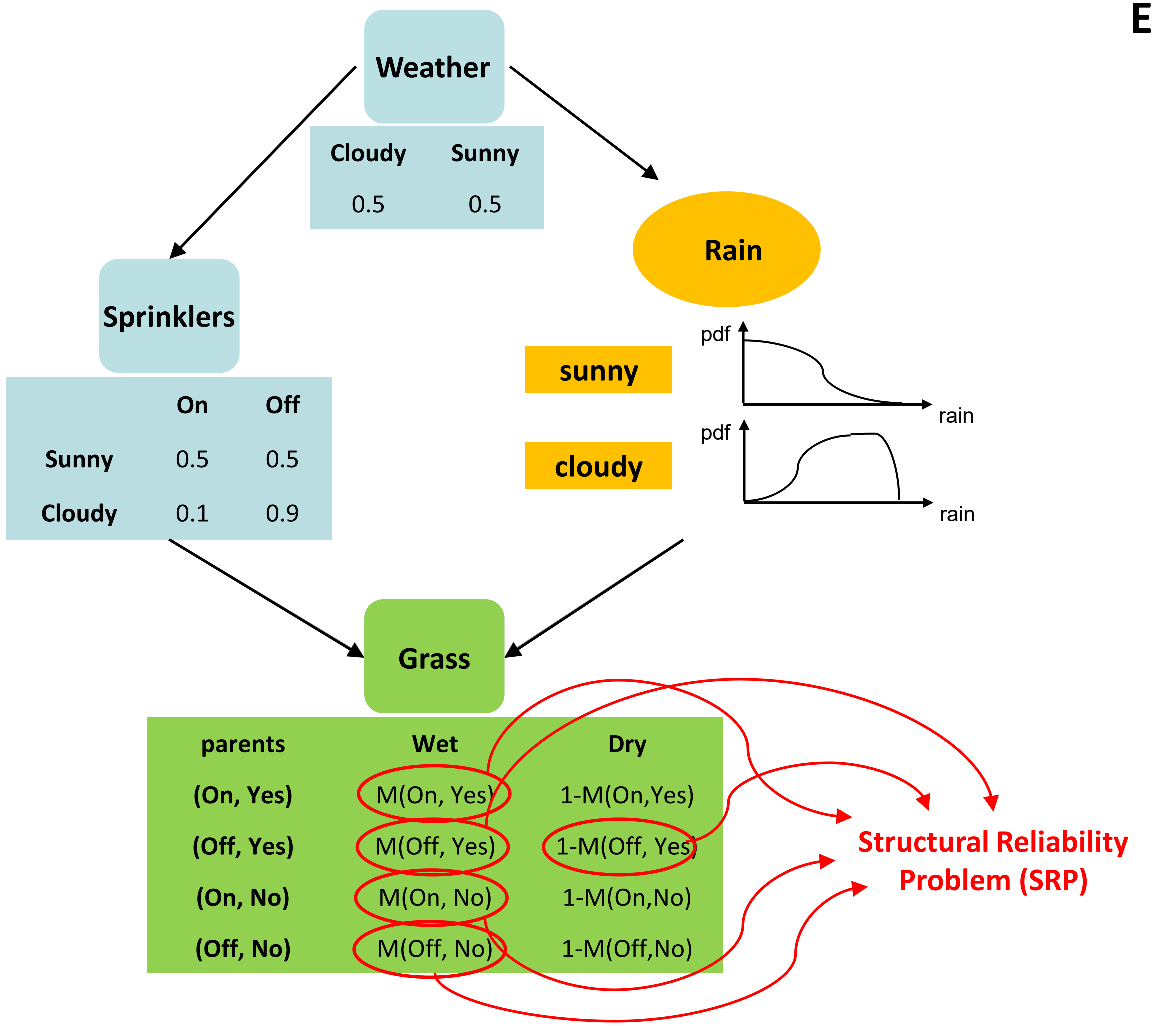
✓ Inference Analysis

✗ **No Continuous Event = No Uncertainties**

✗ **No Small Probabilities**

In the “deterministic” BN’s framework, Functional Nodes CPTs are evaluated through a **single** simulation

Enhanced Bayesian Network



Enhanced Bayesian Network (eBN) [1]

DAG where at least one node is **continuous**

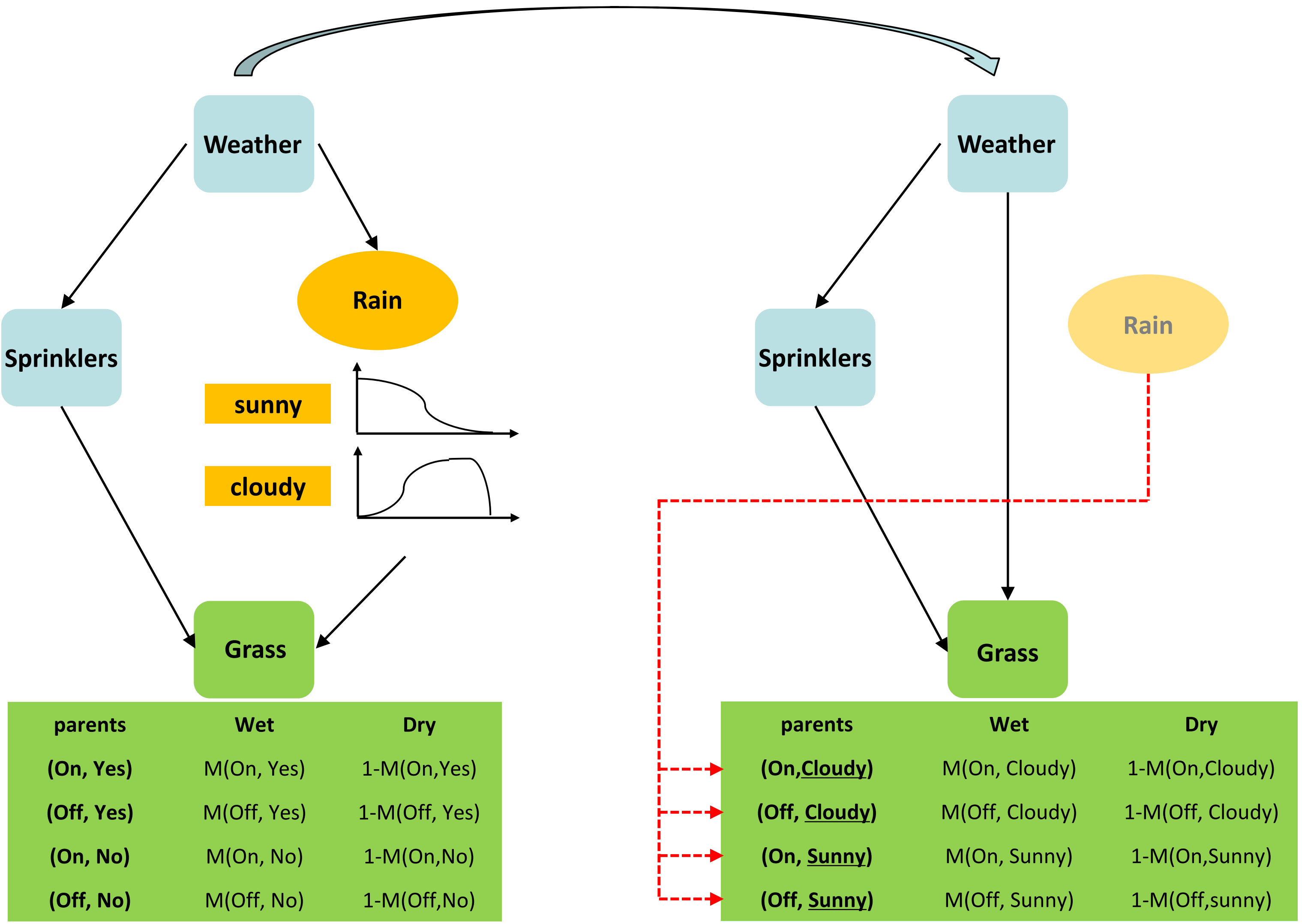
In the eBN's framework, Functional Nodes are evaluated through **multiple** simulations

"Enhanced" = "enhanced with ***structural reliability methods****"* which are used for the CPTs evaluation

- ✓ Different Scenario Analysis
- ✓ Evidence Analysis
- ✓ Yes Continuous Event = Yes Uncertainties
- ✓ Yes Small Probabilities
- ? Inference Analysis

1 D. Straub "Bayesian Network Enhanced with Structural Reliability Methods"

Enhanced Bayesian Network



parents	Wet	Dry
(On, Yes)	M(On, Yes)	1-M(On,Yes)
(Off, Yes)	M(Off, Yes)	1-M(Off, Yes)
(On, No)	M(On, No)	1-M(On,No)
(Off, No)	M(Off, No)	1-M(Off,No)

parents	Wet	Dry
(On,Cloudy)	M(On, Cloudy)	1-M(On,Cloudy)
(Off, Cloudy)	M(Off, Cloudy)	1-M(Off, Cloudy)
(On, Sunny)	M(On, Sunny)	1-M(On,Sunny)
(Off, Sunny)	M(Off, Sunny)	1-M(Off,sunny)

Reduced Bayesian Network (rBN)

Each Continuous Node is **eliminated**

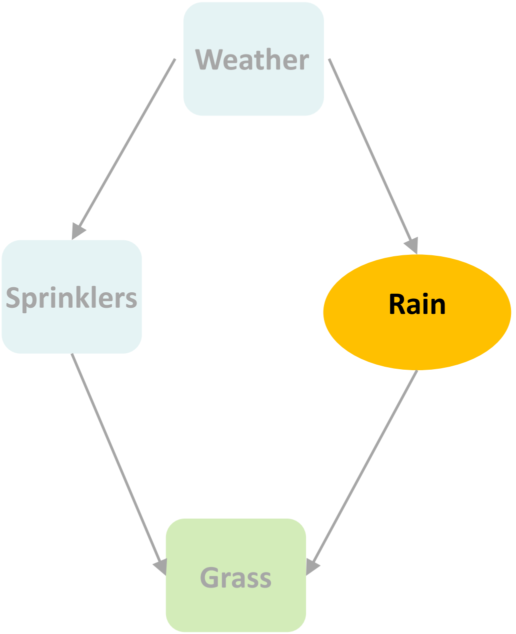
Continuous Nodes with future **evidence** must be **discretized**

Continuous nodes informative content is *transferred* to the network in each SRP

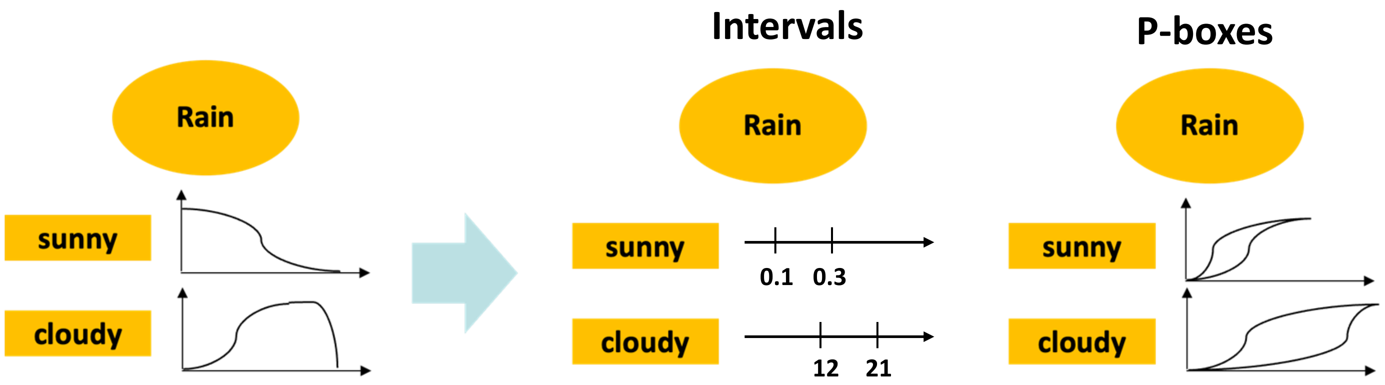
The **rBN** is identical to a **BN**

✓ **Inference Analysis**

Enhanced Bayesian Network – Imprecise



Imprecise Continuous Nodes



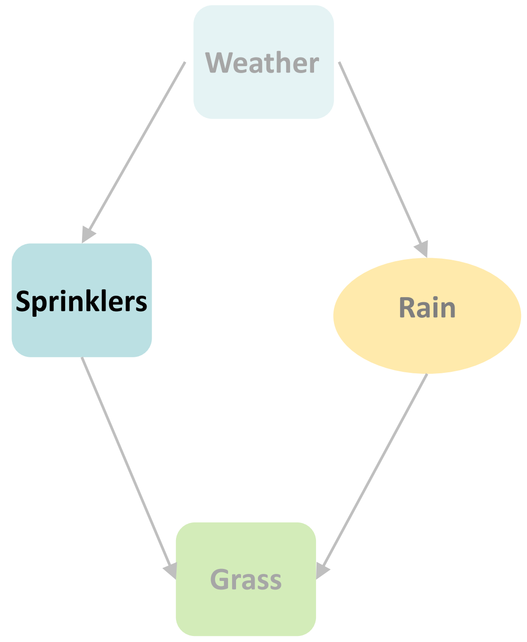
Continuous Imprecise parents

- *failure probability* will be evaluated through a **double-loop** approach

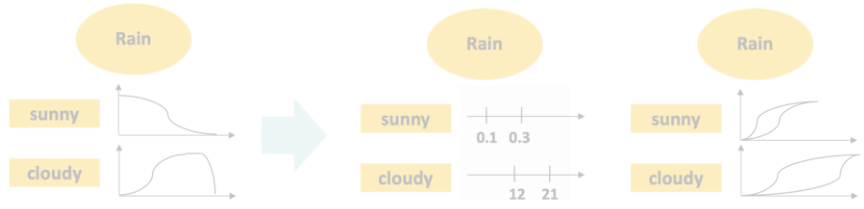
Consequences

- Model node states will be defined through **credal set**
- Classical exact inference algorithm will be no-longer valid

Enhanced Bayesian Network – Imprecise



Imprecise Continuous Nodes



Imprecise Discrete Nodes

Sprinklers

	On	Off
Sunny	0.5	0.5
Cloudy	0.1	0.9

Credal sets

Sprinklers

	On	Off
Sunny	[0.3, 0.5]	[0.5, 0.7]
Cloudy	[0.1, 0.2]	[0.8, 0.9]

Discrete Imprecise parent for a functional node will not affect *failure probability* evaluation

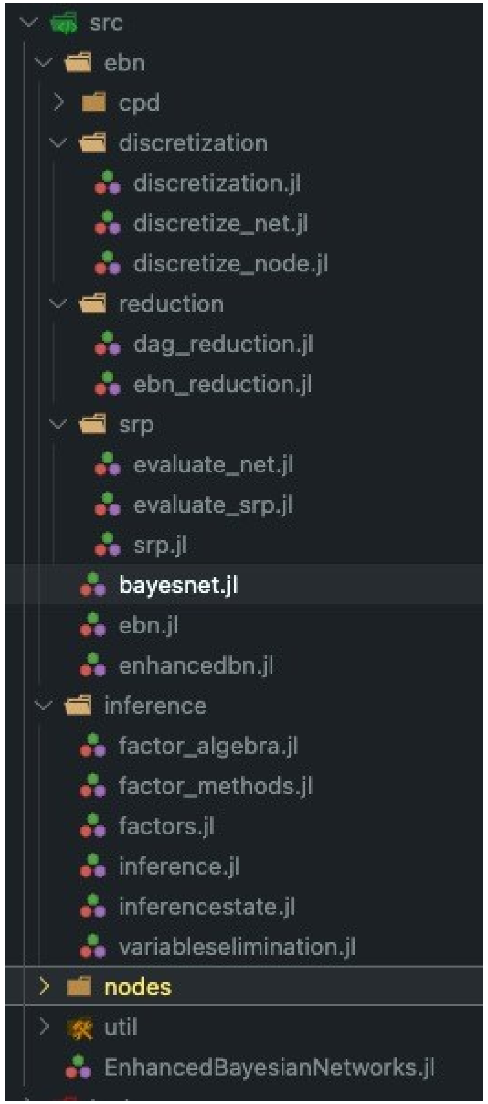
Consequences

- Classical exact inference algorithm will be no-longer valid

Implementation

julia General Porpose, Open Source, MIT, High Performance, Composable.

EnhancedBayesianNetworks.jl



Implementation

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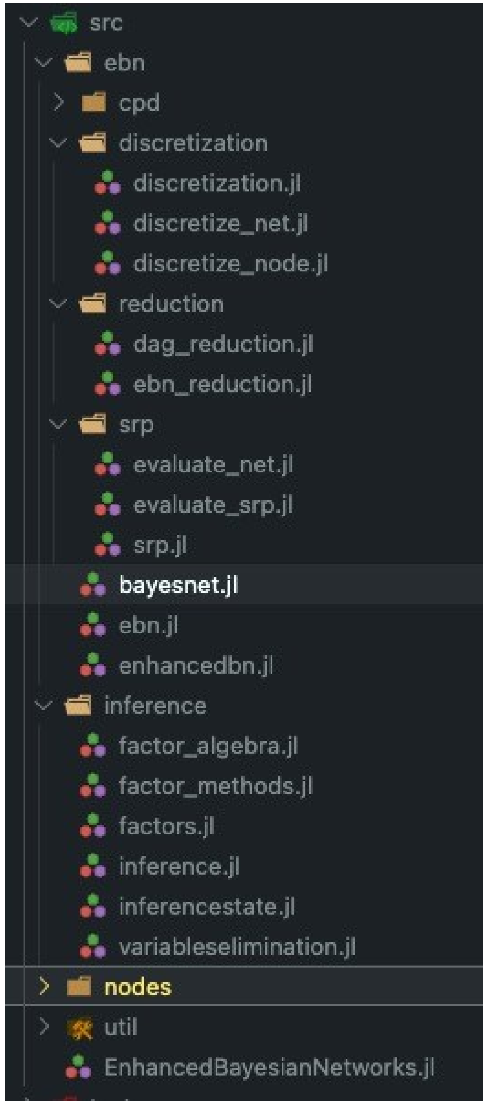
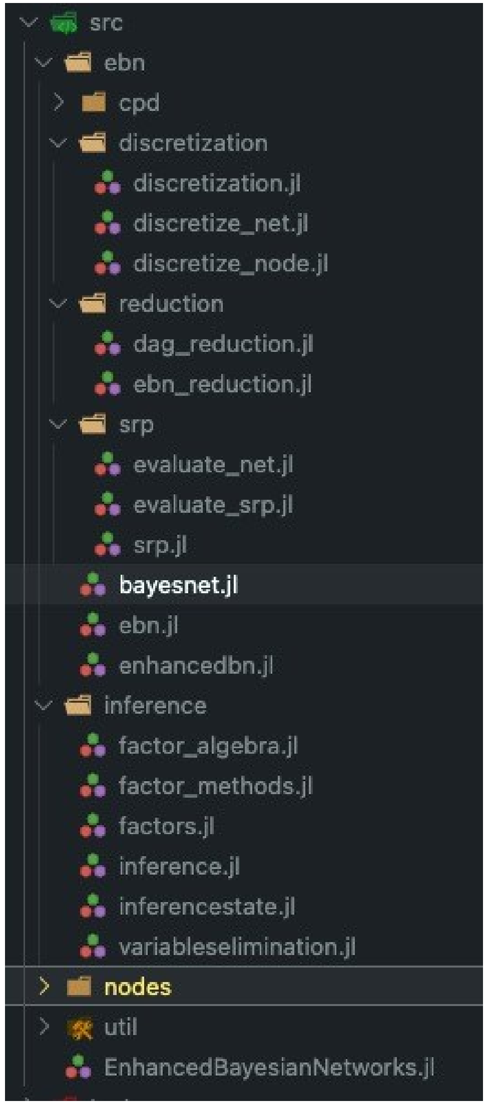


EnhancedBayesianNetworks.jl

- **Root**, **Child** and **Functional** nodes:
  - Both *Continuous* or *Discrete*
  - Both *Precise* or *Imprecise*

Implementation



julia General Porpose, Open Source, MIT, High Performance, Composable.

EnhancedBayesianNetworks.jl

- **Root**, **Child** and **Functional** nodes:
- **Discretization** of each type of Continuous Node

Implementation

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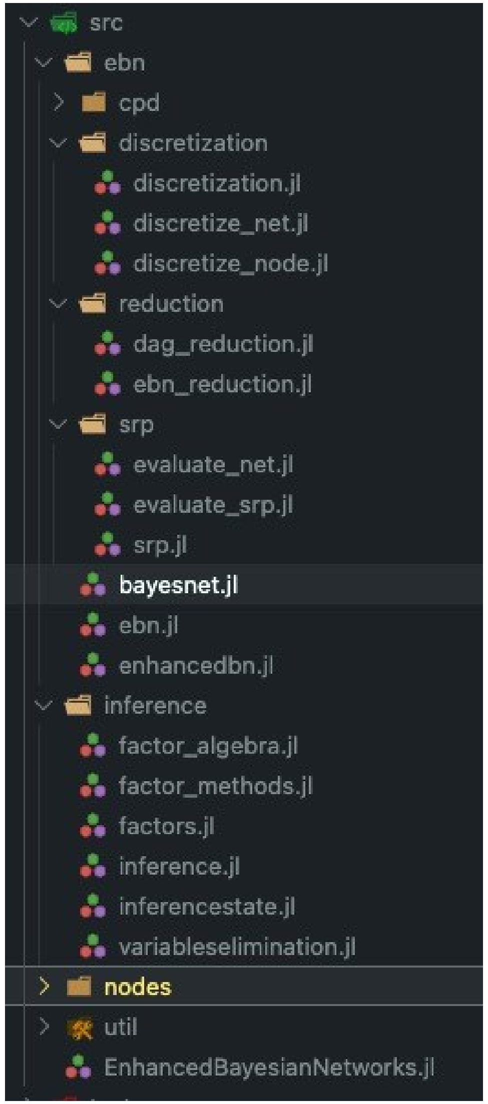
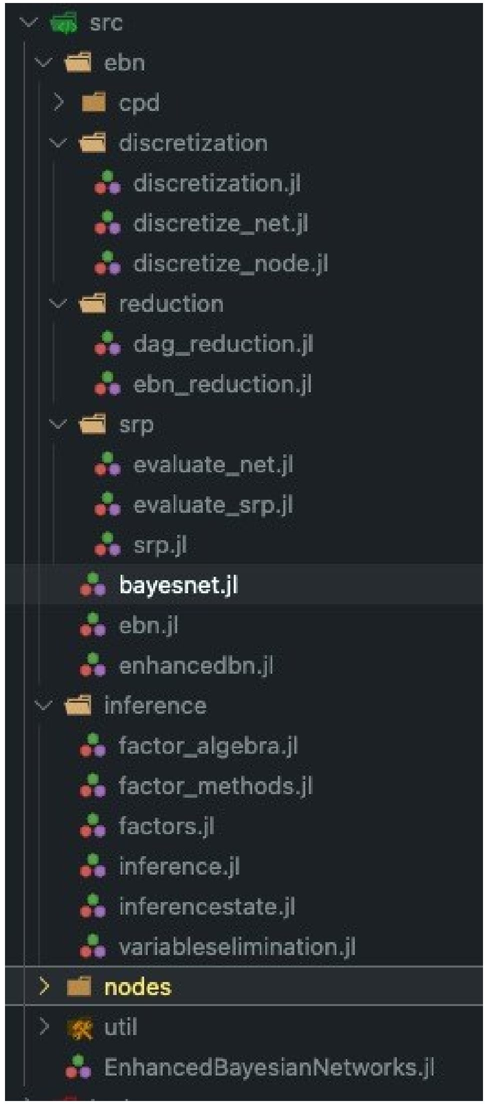


EnhancedBayesianNetworks.jl

- **Root**, **Child** and **Functional** nodes:
- **Discretization** of each type of Continuous Node
- **Multiple Functional Nodes logic**
    - Optimization for error propagation minimization

Implementation

General Porpose, Open Source, MIT, High Performance, Composable.

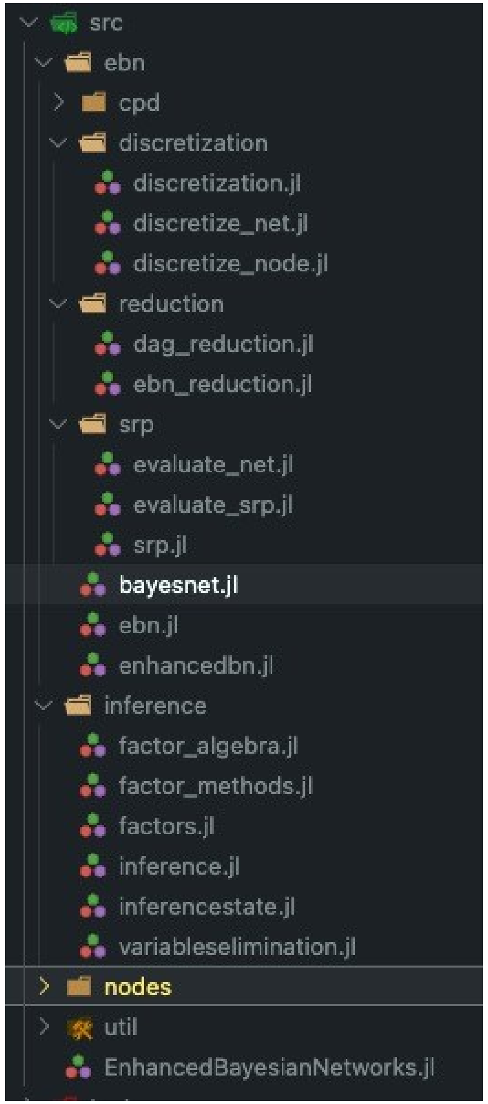


EnhancedBayesianNetworks.jl

- **Root**, **Child** and **Functional** nodes:
- **Discretization** of each type of Continuous Node
- **Multiple Functional Nodes logic**
- Exact **Inference** alghorithm
  - "Minimum Increase In Complexity" optimization

Implementation

julia General Porpose, Open Source, MIT, High Performance, Composable.

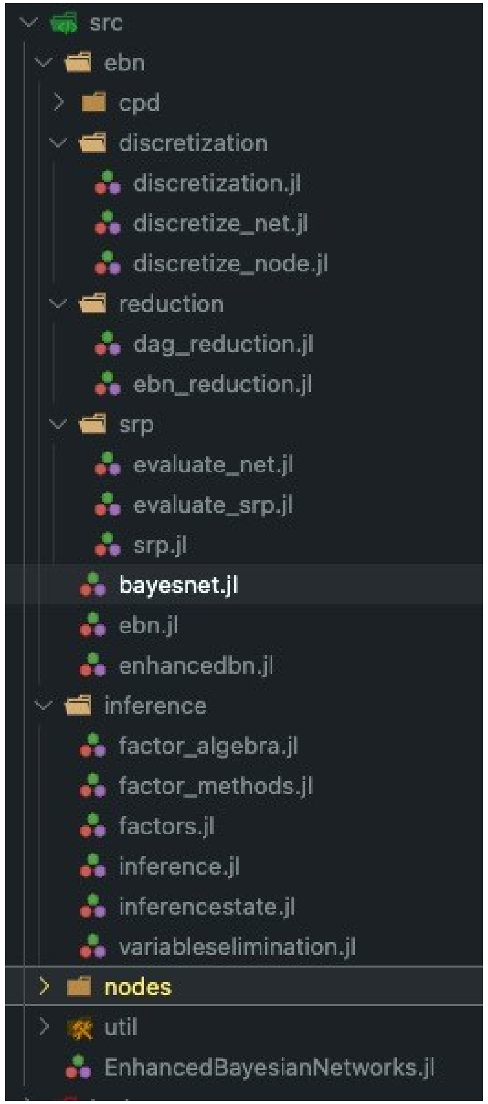


EnhancedBayesianNetworks.jl

- **Root**, **Child** and **Functional** nodes:
- **Discretization** of each type of Continuous Node
- **Multiple Functional Nodes logic**
- Exact **Inference** alghorithm
- General **Node-Elimination** logic for solving the eBN
  - Always ensure acyclicity of the eBN

Implementation

julia General Porpose, Open Source, MIT, High Performance, Composable.

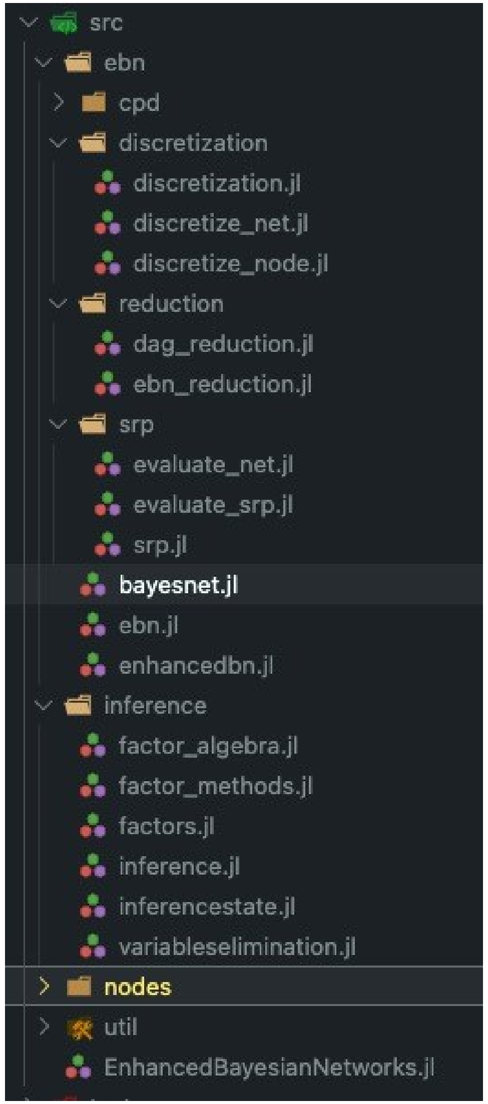


EnhancedBayesianNetworks.jl

- **Root**, **Child** and **Functional** nodes:
- **Discretization** of each type of Continuous Node
- **Multiple Functional Nodes logic**
- Exact **Inference** alghorithm
- General **Node-Elimination** logic for solving the eBN
- **Imprecise probabilities**
  - *Continuous* nodes as both **Interval** and **p-box**
  - *Discrete* node as **credal sets** (WIP)

Implementation

julia General Porpose, Open Source, MIT, High Performance, Composable.



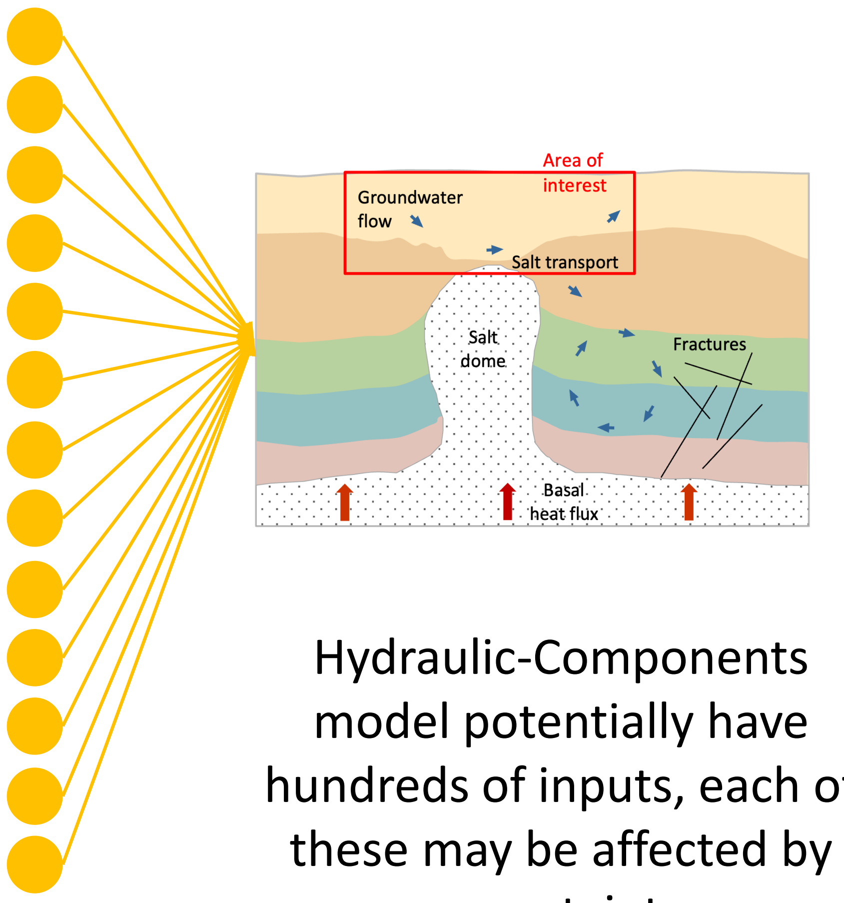
EnhancedBayesianNetworks.jl

- **Root**, **Child** and **Functional** nodes:
- **Discretization** of each type of Continuous Node
- **Multiple Functional Nodes logic**
- Exact **Inference** alghorithm
- General **Node-Elimination** logic for solving the eBN
- **Imprecise probabilities**
- **SRP** can be solved throught **Standard** or **Advanced** *MonteCarlo techniques*
  - Every external model(s) can be used
  - Simulations rely on “**UncertaintyQuantification.jl** ” (“https://github.com/FriesischScott/UncertaintyQuantification.jl”) [2]

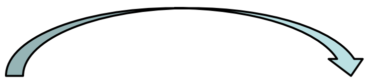
2 J. Behrensdorf “UncertaintyQuantification.jl: A new framework for Uncertainty Quantification in Julia”

Salt Dome

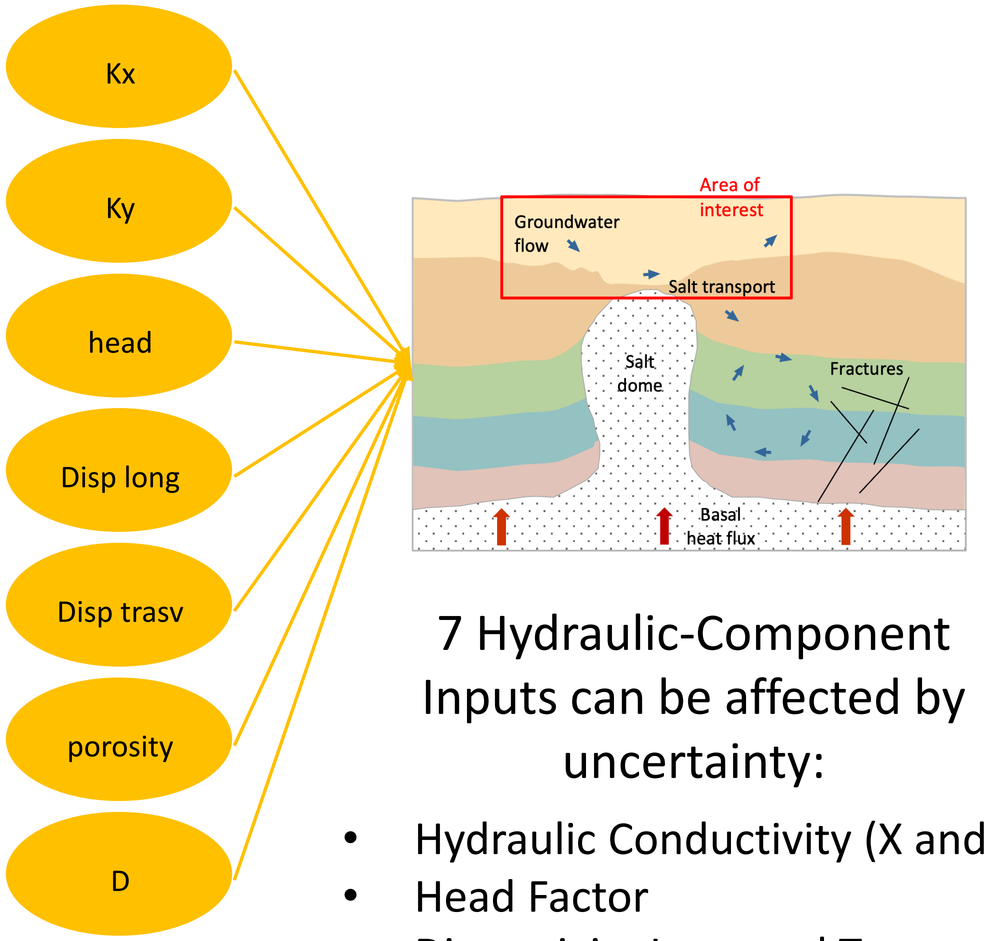
Salt Dome – Model Inputs



Hydraulic-Components model potentially have hundreds of inputs, each of these may be affected by uncertainty



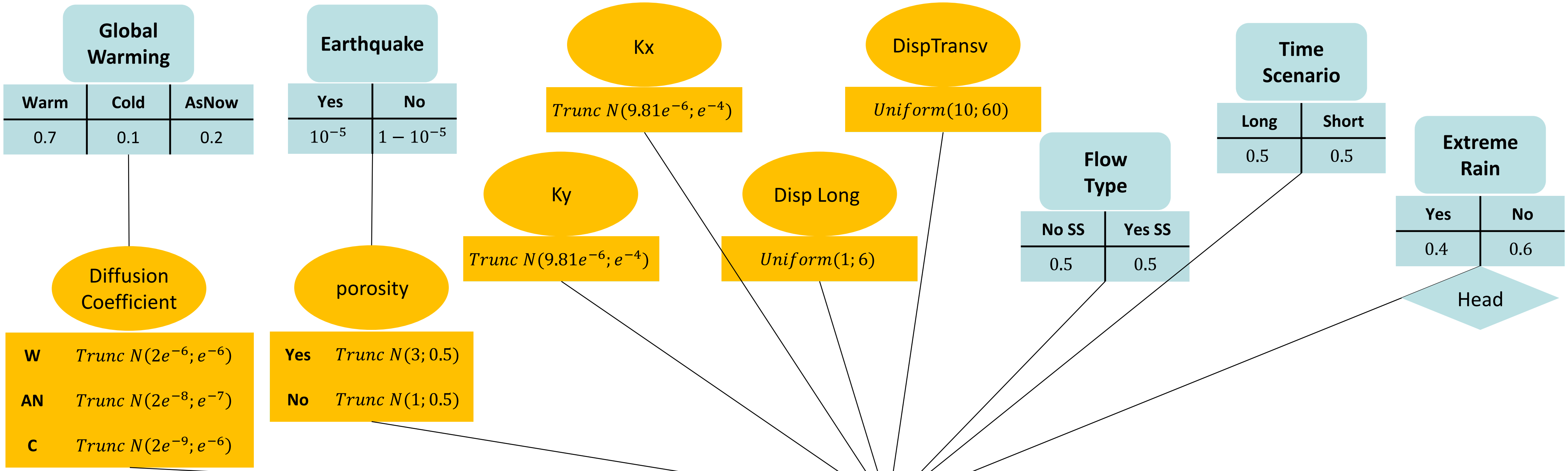
Expert based knowledge



7 Hydraulic-Component Inputs can be affected by uncertainty:

- Hydraulic Conductivity (X and Y)
- Head Factor
- Dispersivity Long and Transv
- Porosity
- Diffiusion Coefficient

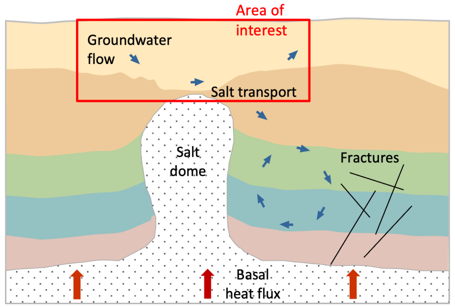
Salt Dome – eBN



Salt Dome eBN - "Toy Example"

Model:

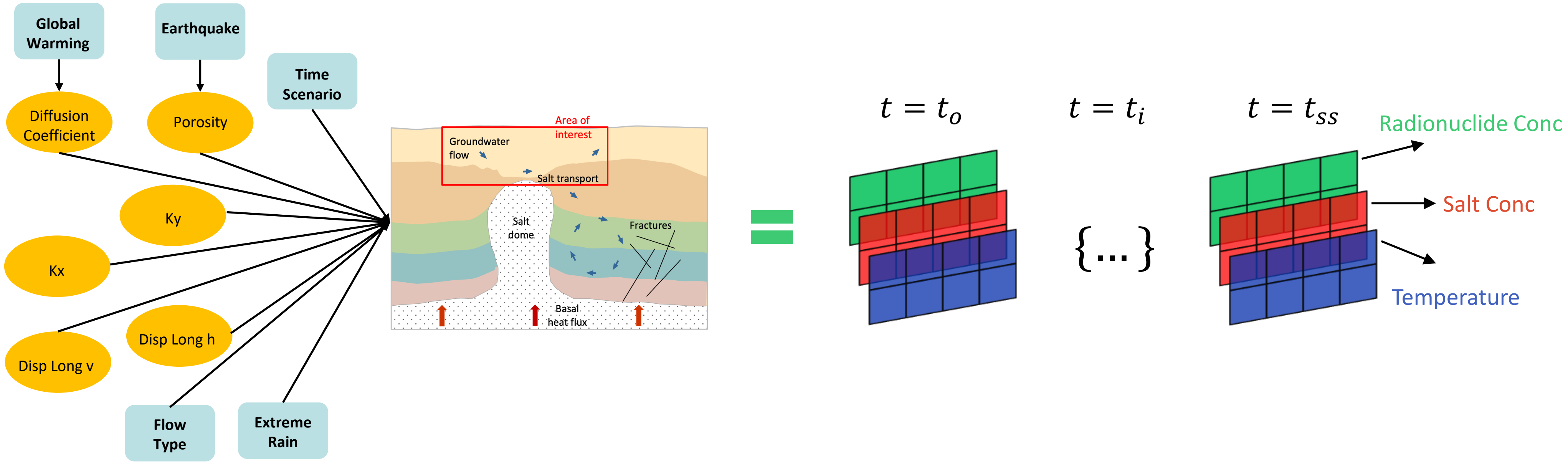
- FE model (60x30)
- No heat source
- Salt and Radionuclide Transport
- Density driven flow simulation



Nodes definition and relationships are steps based on expert knowledge

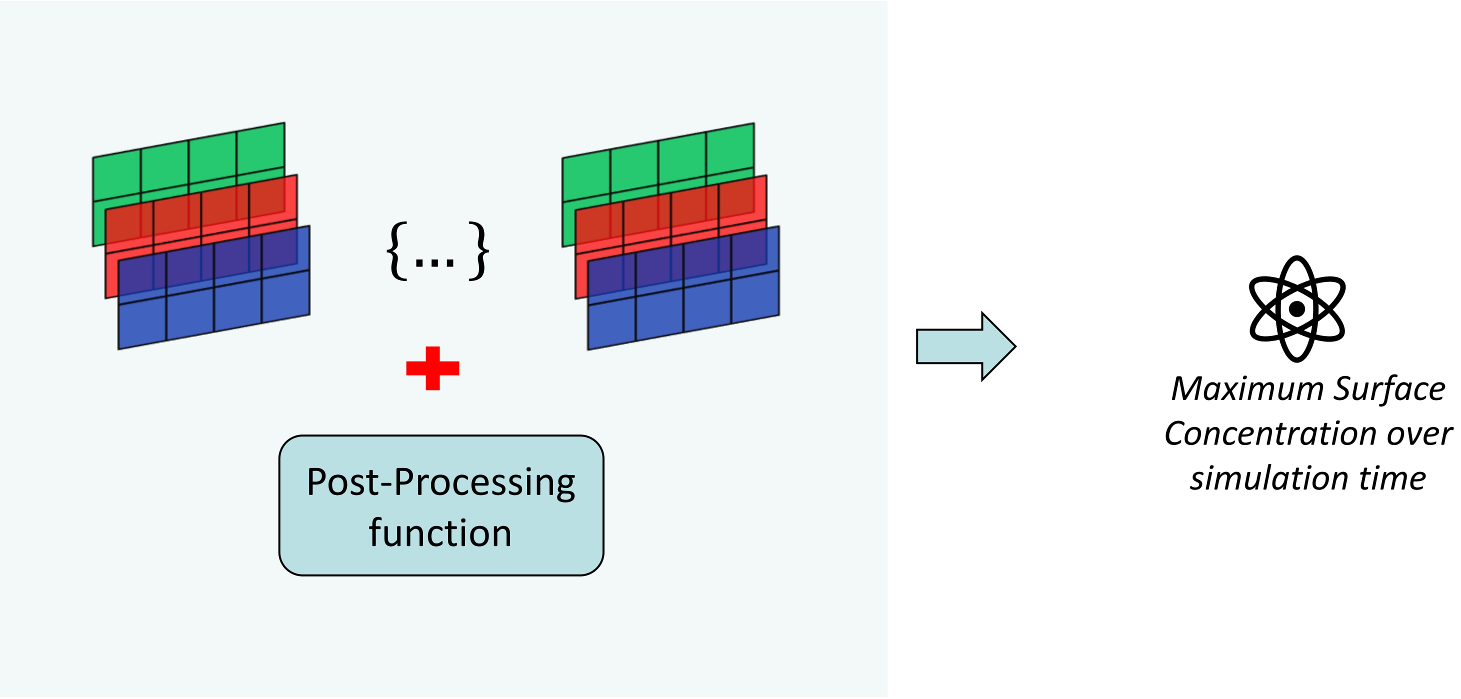
The presented eBN and its node do **NOT** represent any real case application

Salt Dome – Model Output



- The HC Finite Elements model returns 3 (60 x 30) matrices at each time stamp
- In order to perform a Reliability Assessment, a **Performance function** is needed!

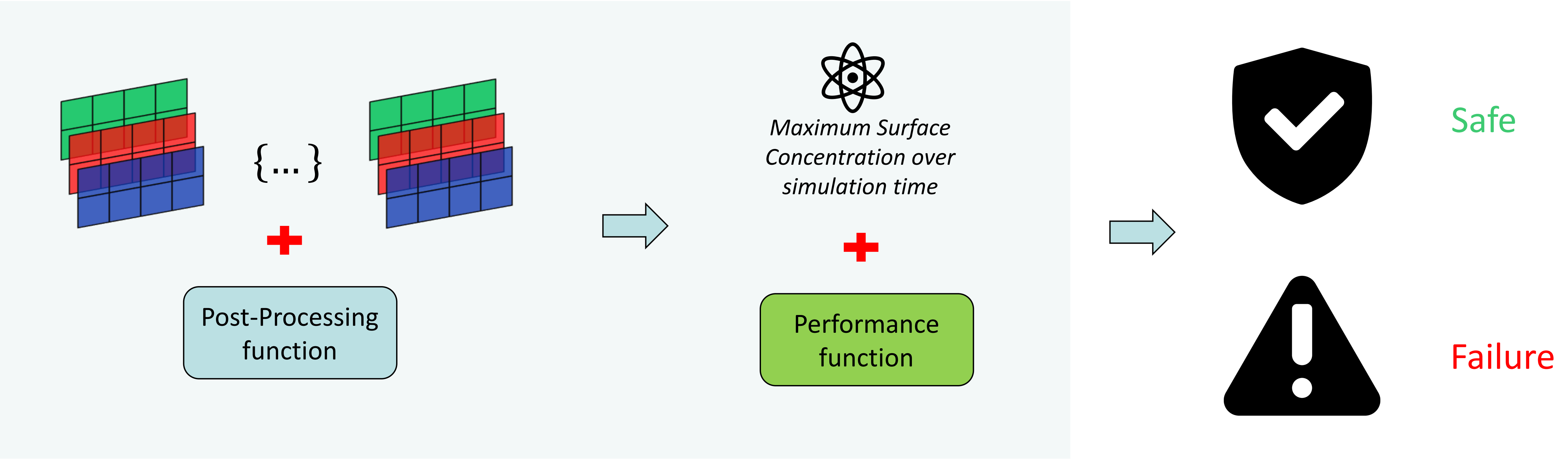
Salt Dome – Model Output



- **Post-Processing Function**
  Collapse the information to a single value

- **Maximum surface concentration over simulation time**
  Maximum radionuclides concentration value in the first 20 layers

Salt Dome – Model Output

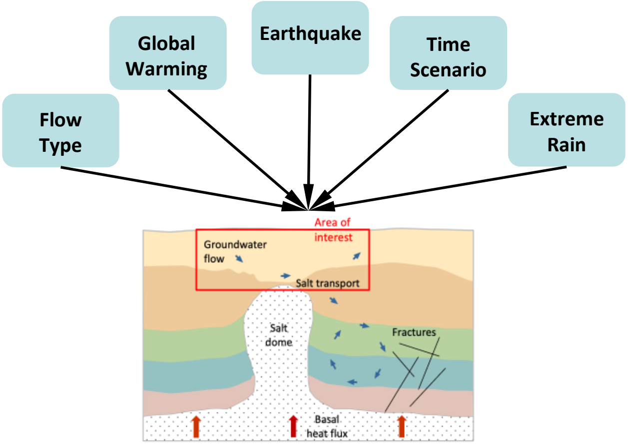


- **Performance Function**
  Definition of the *Failure* and *Safe* System's state

- In this case we choose a very strict performance:
  Threshold = 0

Salt Dome – Results

rBN – THC model's Node



- Extreme Rain => 2 states
- Time Scenario => 2 states
- Flow type => 2 states
- Earthquake => 2 states
- Global Warming => 3 states

$2^4 * 3 = 48$ SRPs (200 simulations each)

Performance => Surface Concentration > 0

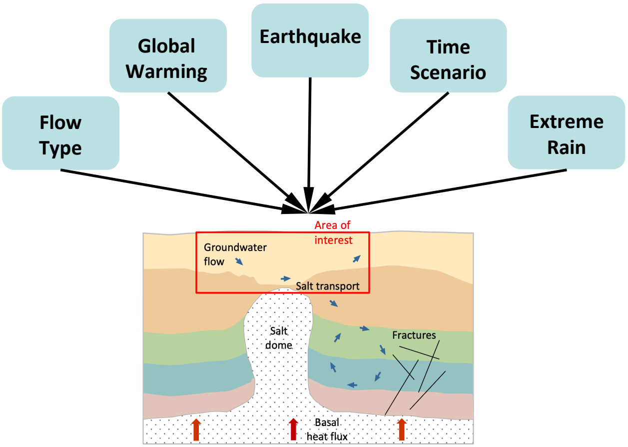
State	Failure	Safe
comb 1	0.995	0.005
comb 2	1	0
comb 3	0.87	0.13
comb 4	0.885	0.114
comb 5	0.92	0.079
comb 6	0.875	0.125
comb 7	1	0
comb 8	0.83	0.17
comb 9	0.985	0.015
comb 10	0.99	0.01
comb 11	0.885	0.114
comb 12	0.92	0.079

{ … }

State	Failure	Safe
comb 22	0.985	0.015
comb 23	0.94	0.06
comb 24	0.98	0.02
comb 25	1	0
comb 26	1	0
comb 27	0.985	0.015
comb 28	0.82	0.18
comb 29	0.93	0.069
comb 30	0.835	0.165
comb 31	0.905	0.094
comb 32	0.895	0.104
comb 33	0.905	0.094

Salt Dome – Results

rBN – Inference



Time Scenario

$$\begin{cases} p(time\_long|HC\_fail) = 0.57 \\ p(time\_short|HC\_fail) = 0.43 \end{cases}$$

Earthquake

$$\begin{cases} \boldsymbol{p(earthquake|HC\_fail) = 10^{-5}} \\ p(no\_earthquake|HC\_fail) = 0.99999 \end{cases}$$

Extreme Rain

$$\begin{cases} p(er|HC\_fail) = 0.54 \\ p(no\_er|HC\_fail) = 0.46 \end{cases}$$

Global Warming

$$\begin{cases} p(cooling|HC\_fail) = 0.11 \\ p(astoday|HC\_fail) = 0.19 \end{cases}$$

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- Establish a **conversion method from FT to BN** for exploiting the extensive litterature regarding Risk Assessment of complex system with FT