Seismic Forward Modelling to Characterize Spatial Uncertainties of Geological Structures

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Introduction

Spatial uncertainty is critical in the design of underground nuclear waste storage sites as they must comply with specific safety distances to the boundaries of the selected geological structure to hold the site. This uncertainty is larger in areas where data is not available and interpolation among measured data is needed. We propose a method to identify the optimal variogram parameters from available seismic data. Starting from the seismic interpreted model of acoustic impedances, we produce perturbations by adding several realizations of a Gaussian field (produced with different range, sill and nugget parameters). We simulate their corresponding seismic data with forward modelling. By computing the error between the simulated and available seismic data, we are able to obtain the optimal variogram parameters to model the geological structure and use them to characterize its spatial uncertainty with sequential Gaussian simulation. The objective of the method is expected to identify the safest locations (if any) to place a storage site and, especially, characterize the spatial uncertainty between known measurements.

Method

1 INPUT DATA

The analysis starts with the seismic data of the subsurface region whose uncertainty we expect to characterize. We call these data "real seismic data". At this stage of the study, we consider a 2D synthetic seismic section, in depth domain. We interpret the horizon it represents and create an acoustic impedance model.

$$\text{Seismic data} = \boldsymbol{S_{real}}\,(\mathbf{d})$$

$$\text{Acoustic Impedance} = \boldsymbol{M_{real}}\,(\mathbf{d})$$

where $\mathbf{d}$ is a point $(n_d, n_x) \in \mathbb{R}^2$; with $n_d = n_x = 0, 1, \ldots, 99$

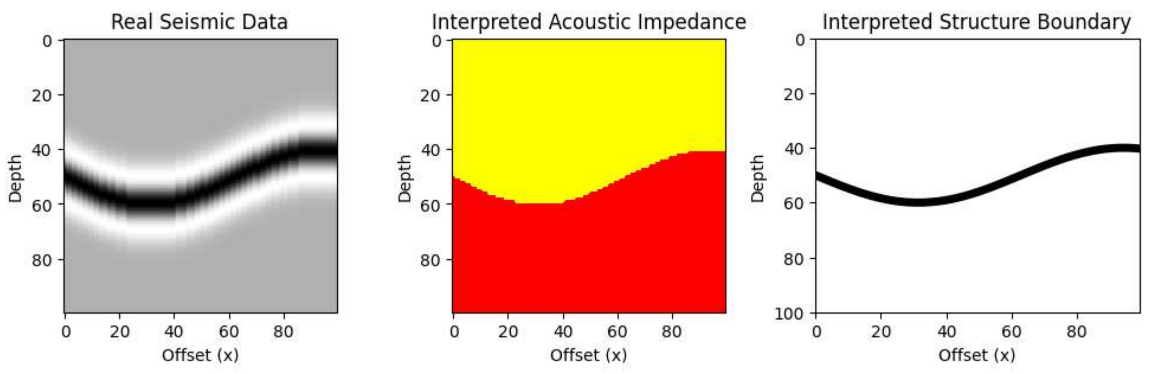

Fig. 1: Synthetic seismic data (left) from the location whose uncertainty is expected to be characterized; along with its corresponding impedance model (middle) and horizon (right).

2 FORWARD MODELLING

By selecting n_m (with $n_m = 10000$ for least-squares and $n_m = 500$ for optimal transport errors, Sambridge et al., 2022) uniformly distributed random samples of variogram parameters (range, sill and nugget), we create n_m realizations of a Gaussian field (with zero mean) and add them to the interpreted acoustic impedance model in step 1. We obtain n_m additional impedance models, whose corresponding seismic data (band-limited post-stack) are generated with forward modelling (PyLops Development Team, 2023).

$$\boldsymbol{S_i} = \mathbf{W}\boldsymbol{S_i}\,(\mathbf{d}, \theta = 0) = w(\mathbf{d}) * \frac{d\,ln\,\boldsymbol{M_i}(\mathbf{d})}{d\mathbf{d}}$$

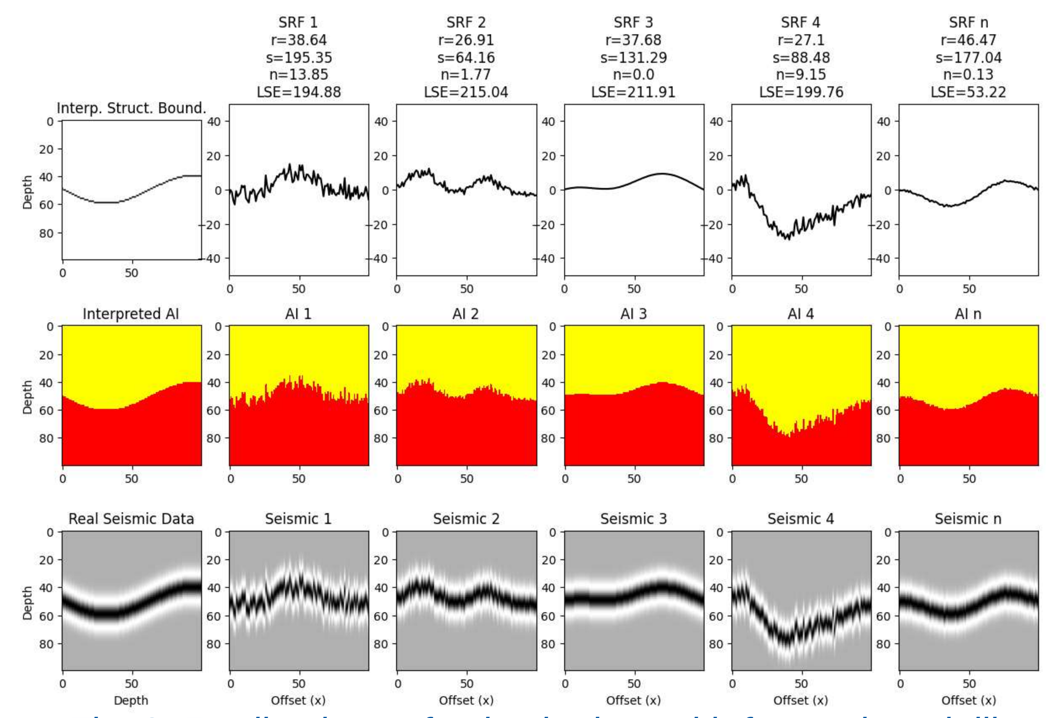

Fig. 2: Realizations of seismic data with forward modelling

3 ERROR MEASURE

By measuring the error between the real seismic section (step 1) and the n_m forward-modelled realizations, we assign a likelihood value to each of them. After testing several methods, we use least-squares and optimal transport (Sambridge et al., 2022) to measure the errors. We obtain the probability distributions for each of the variogram parameters.

$$\text{Least Squares Error} = \frac{1}{2}\sum_{i=1}^{N=n_m}\left(\boldsymbol{S_{real}}(\mathbf{d}) - \boldsymbol{S_i}(\mathbf{d})\right)^2$$

$$\text{Optimal Transport Error} = \int_D \left|\mathbf{ICDS_{real}} - \mathbf{ICDS_i}\right| d\mathbf{d}$$

where $\mathbf{ICDS_{real}}$ *and* $\mathbf{ICDS_i}$ *are the inverse cumulative distributions functions of the real and forward modelled seismic data (Sambridge et al., 2022)*

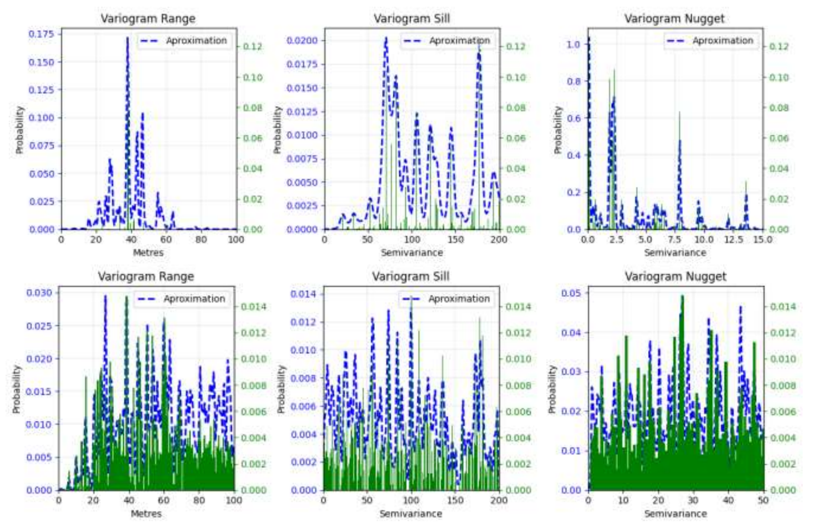

Fig. 3: Distributions of variogram parameters for least-squares (above) and optimal transport (Sambridge et al., 2022)(bottom).

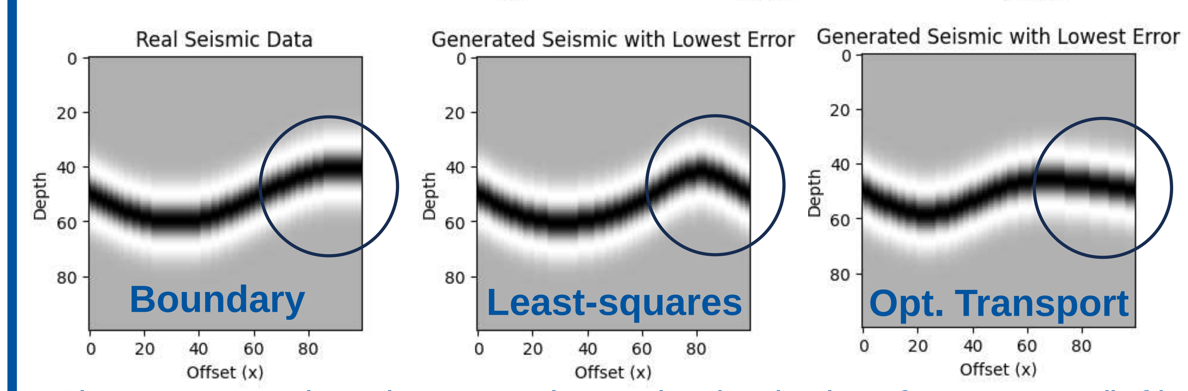

Fig. 4: Comparison between the real seismic data from step 1 (left) and the best forward-modelled seismic data for least-squares (middle) and optimal transport errors (Sambridge et al., 2022)(right)

4 VARIOGRAM MODEL

$$\gamma(\mathbf{d}) = \text{nugget} + (\text{sill} - \text{nugget})\left(1.5\left(\frac{\mathbf{d}}{\text{range}}\right) - 0.5\left(-\frac{\mathbf{d}}{\text{range}}\right)^3\right)$$

$$if\;\; \mathbf{d} < \text{range},\;\; \gamma(\mathbf{d}) = \text{sill}\;\; \text{otherwise}$$

We construct the variogram models from the distributions of the parameters found in previous step and used them in the next step for a sequential Gaussian simulation.

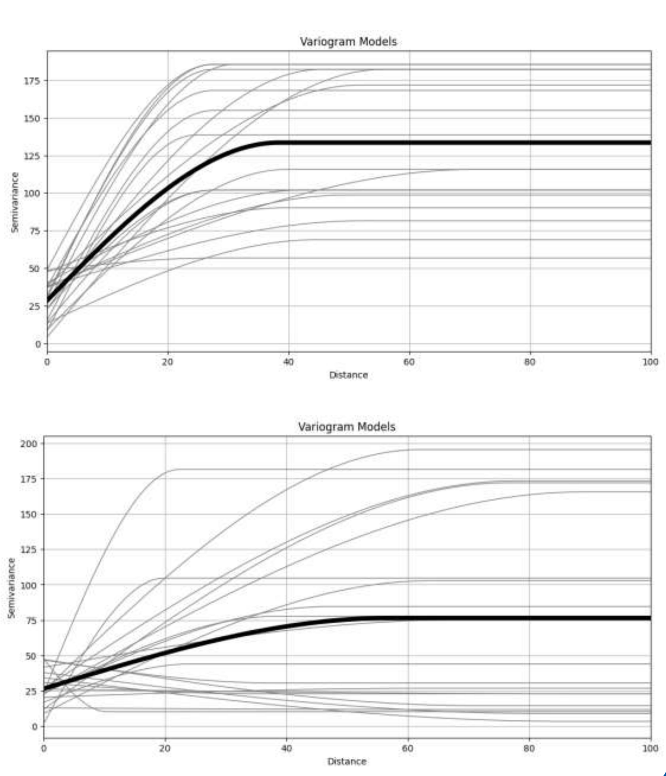

Fig. 5: Modelled variograms, from least-squares (above) and optimal transport (Sambridge et al., 2022)(bottom). Averages shown in bold lines.

5 CHARACTERIZATION OF SPATIAL UNCERTAINTY WITH SEQUENTIAL GAUSSIAN SIMULATION

With the modelled variogram we perform sequential Gaussian simulation to characterize the uncertainty of the area, given the real seismic data in step 1.

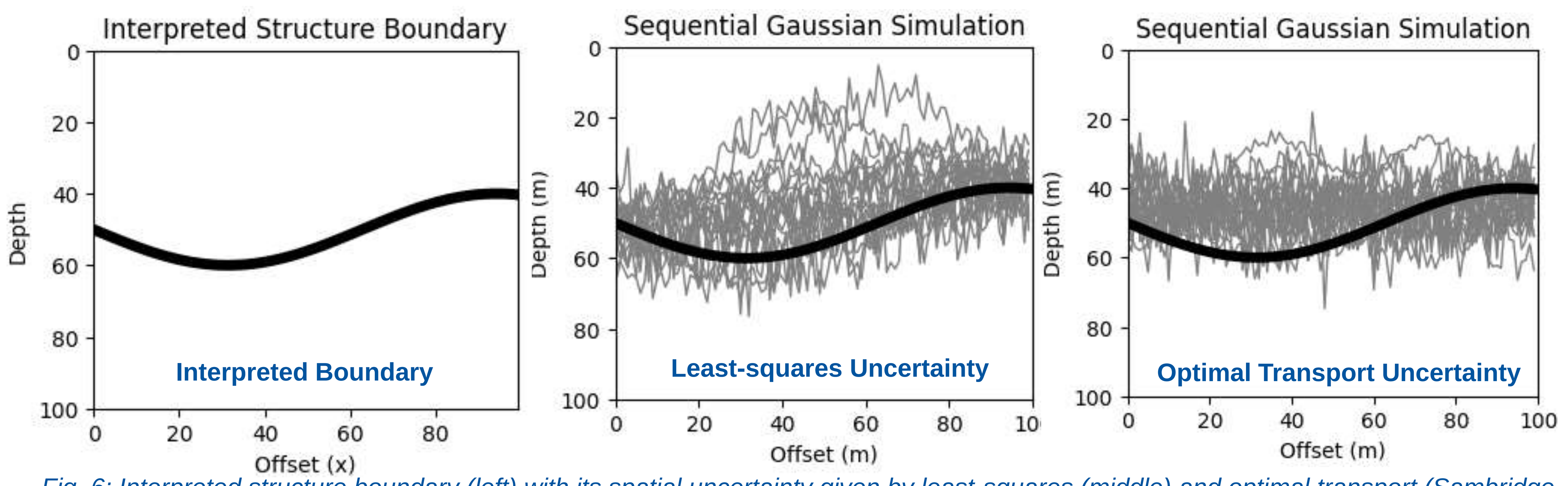

Fig. 6: Interpreted structure boundary (left) with its spatial uncertainty given by least-squares (middle) and optimal transport (Sambridge et al., 2022)(right).

Discussion & Future Work

- Current work has already involve a real 2D section of seismic data.
- The distributions of the variogram parameters show periodicity that needs further investigation.
- Optimal transport provides a more detailed error calculation, however its results are less stable (even with a closer model) and the uncertainty does not follow the interpreted boundary; further investigation is needed.
- The range needs to be of similar magnitude as the width of the section, as larger values can produce similar, non-unique perturbations, resulting in many range values providing indistinguishable results.

References

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