Risk-based Assessment of Salt Domes as Disposal Sites for Nuclear Waste (RADON)

Estimation of risks of pollutant dispersion with Enhanced Bayesian Networks

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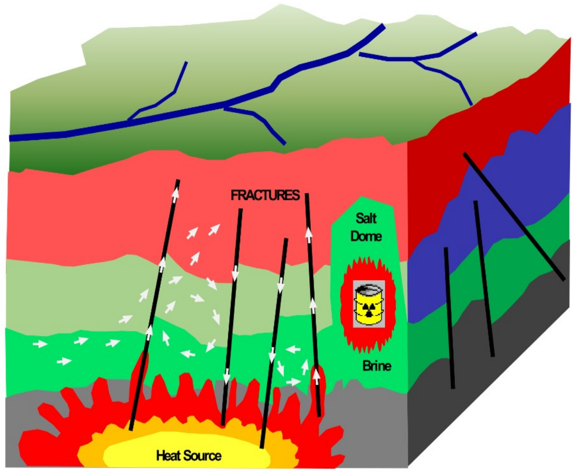
RADoN Project
Motivation

- Final disposal of high level nuclear waste (heat generating) in deep geological formations
- Isolating waste from biosphere
- Safety time span of 1 million years
- 3 possible host rocks: clay, cristalline and salt rock
- This project is focussing on salt domes
- Salt rock (salt domes) have been investigated intensively in Germany (Gorleben)

RADoN Project

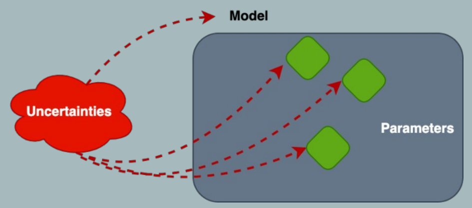


Quantitative *risk assessment* which takes into account the combined effect of

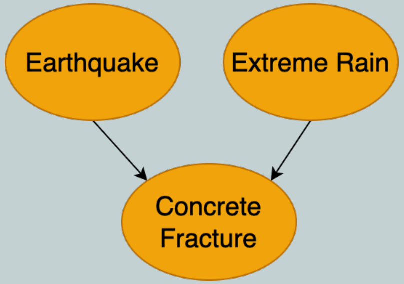
All relevant **sources of uncertainty**

e.g.

- subsurface structure
- material properties
- BCs

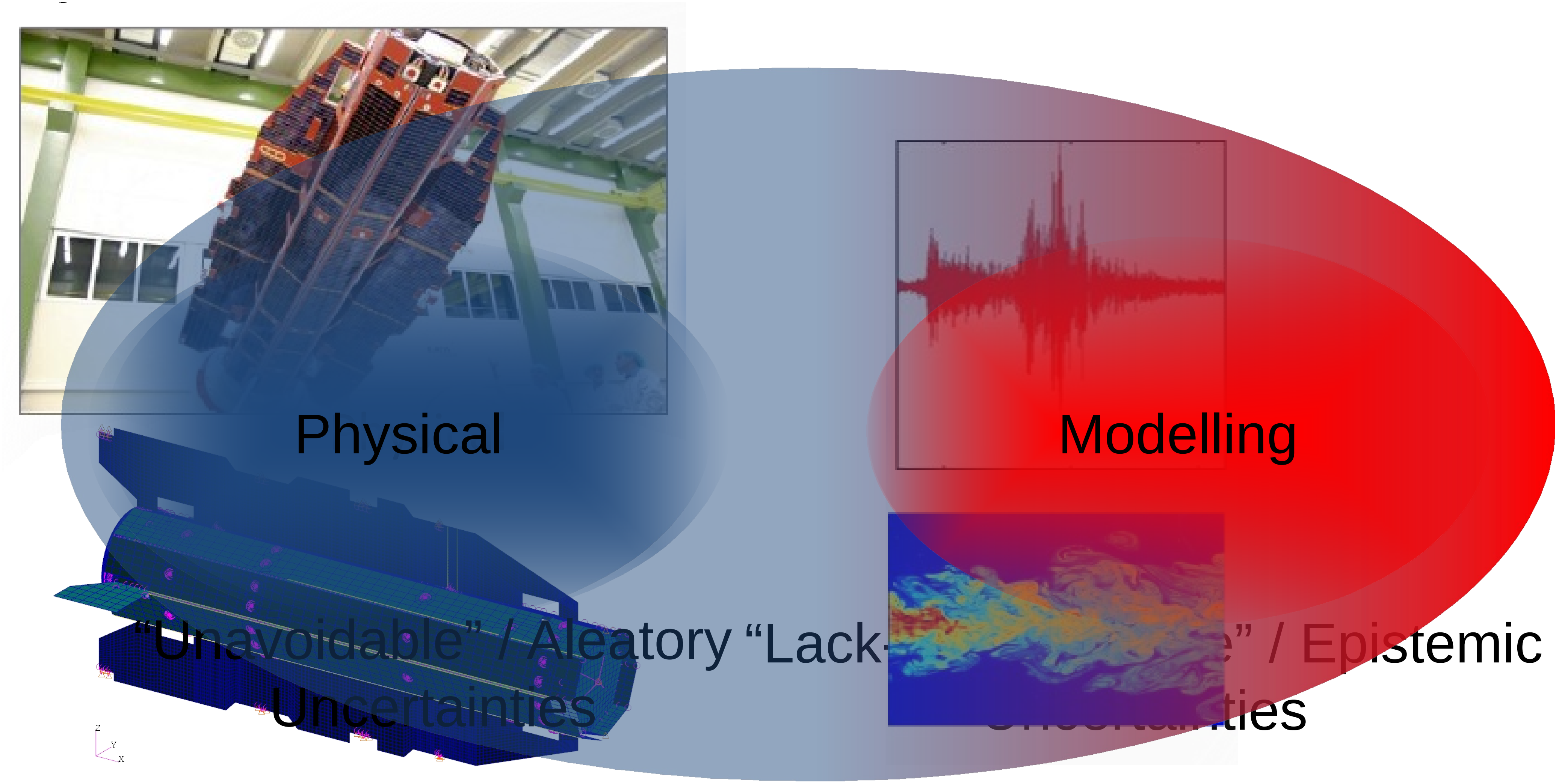


Hazardous events with their probability and dependencies

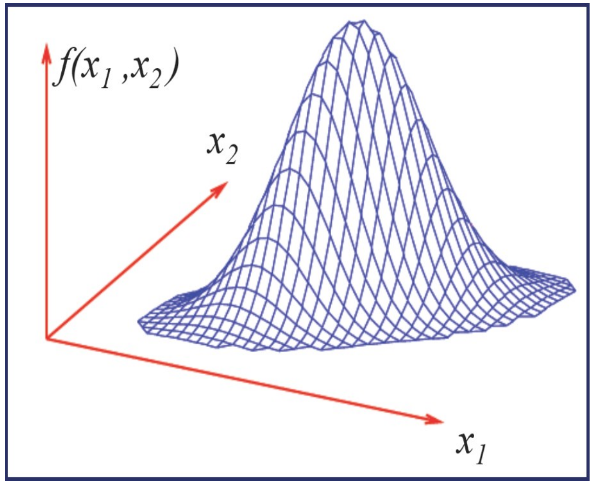


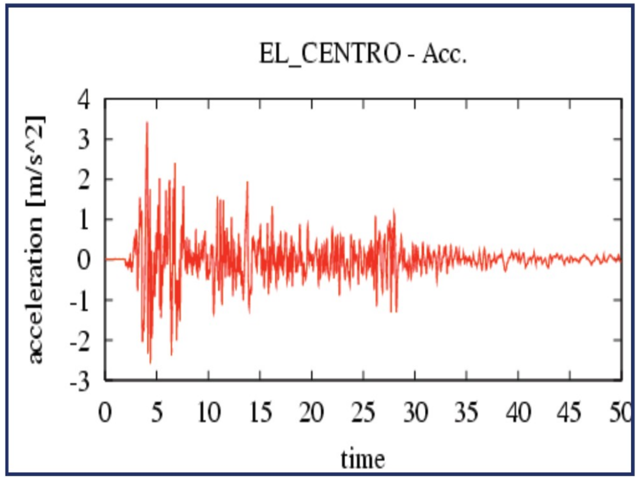
- ➢ One of the most *challenging* part is to identify **the hazardous (and not hazardous) events** and describe their relationships (CDFs).
- ➢ This process is **expert-knowledge based.**
- ➢ The model must be *able to update* all the assumptions on hazardous events CDFs whenever data became available.

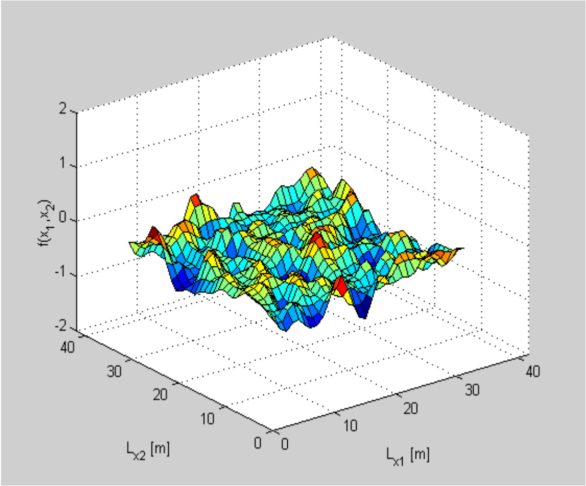
Spectrum of uncertainties



Aleatory uncertainties



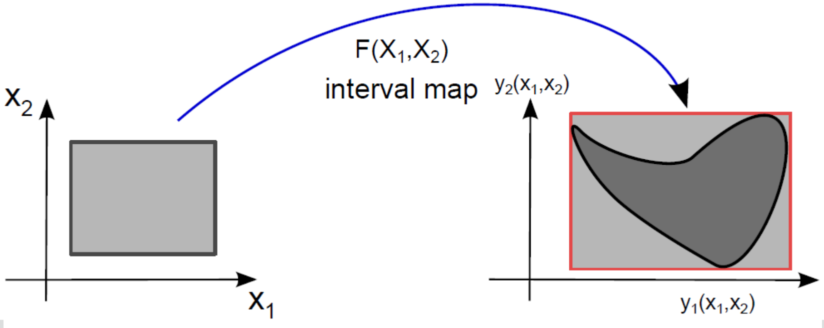


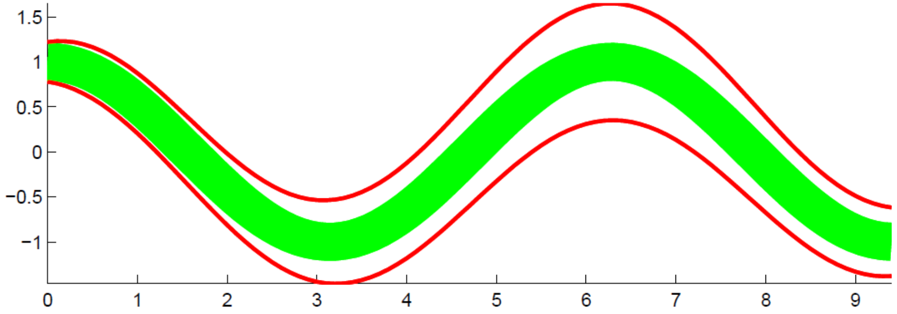


- Random variables
  - Random parameters, e.g., spring stiffness, dimension, static load
- Stochastic processes
  - Random functions of time e.g. Excitation time history, earthquake, dynamic load
- Random fields
  - Spatially fluctuating properties e.g. Young's modulus, Shell thickness

Epistemic uncertainties

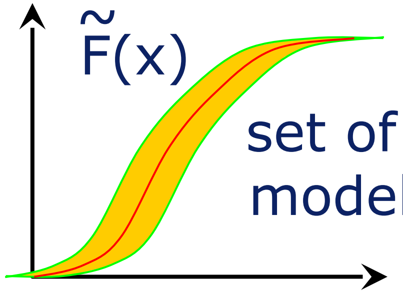
- Statistical information often not available
  - e.g. unique structure
- Lack of knowledge
  - e.g. few or missing data
- Qualitative information
  - e.g. expert judgements





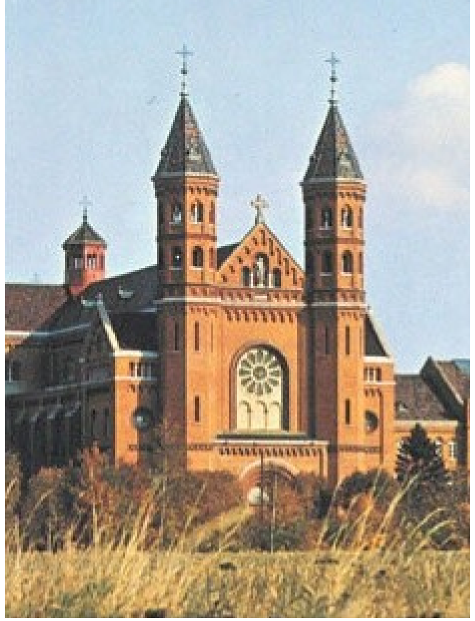
Vague and imprecise information

statistical analysis of imprecise and rare data



set of plausible models

Is it safe ?



reliability analysis

$\tilde{P}_f$

$[P_{f,l}, P_{f,r}]$

imprecision reflected in P_f

Is the reliability analysis still reliable ?

Effects on P_f ?

Sensitivity of P_f to imprecision ?

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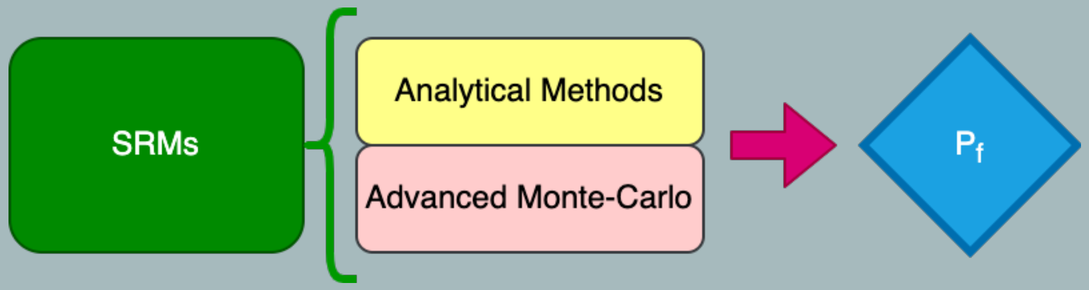
Bayesian Networks Enhanced with Structural Reliability Methods

Structural Reliability Methods

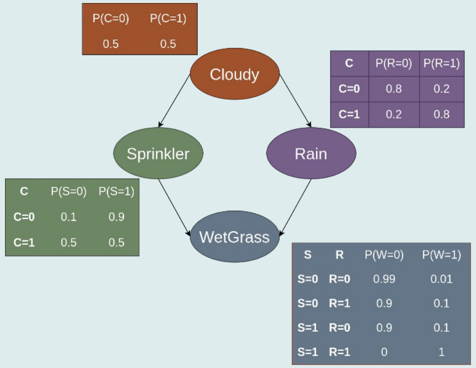
- System's state can be:
  - '***safe***' with a given set of parameters
  - '***not safe***' with a **slightly changed** set of parameters

- When one (or more) parameter/s of the system are **affected by uncertainties**:
  - *Parameter*/s becomes *random variable/s* ''
  - *System state* became **dependent** on *random variable/s*

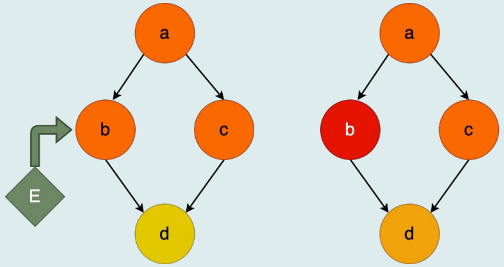


Bayesian Networks

- For evaluating the reliability of a system in **different scenarios**
- General features:
  - *multidisciplinary-usability*
  - *Human-readbility*
  - *Compact-representation*

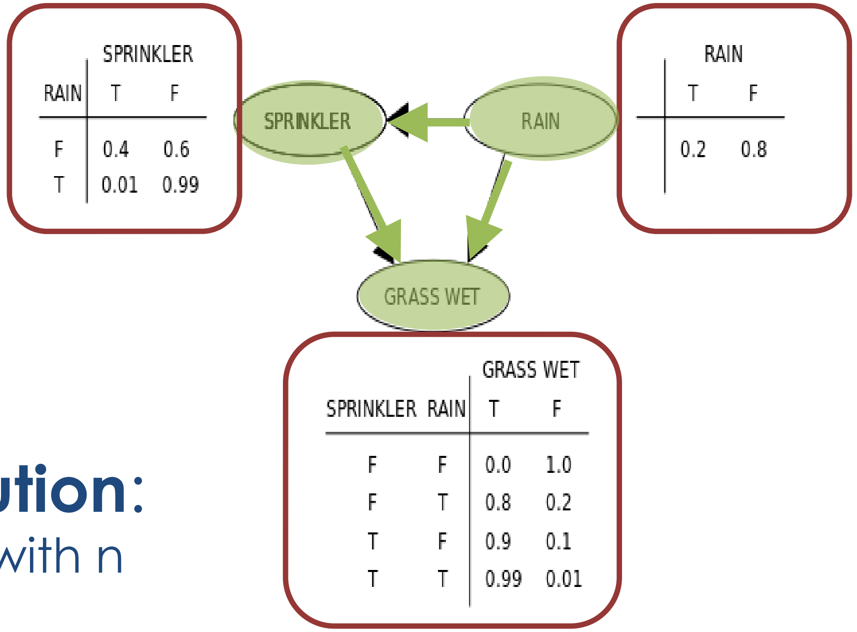


- Specific featurse:
  - **Bayesian Update** of marginal probabilities (once new data 'E' becames available)
  - **what-if** *analysis*
  - ***propagation*** *of the information on the direction of interest*



Bayesian Networks - Structure

- **Nodes**
  - **Parent:** node influencing others
  - **Root:** node with no parents
- **Edges**
  - Causal dependencies
  - Local **Markov property** verified
- **Conditional Probability Distribution**:
  - Discrete: **CPT** (2^{n+1} entries if Boolean with n parents)
  - Continuous: Normal or Gaussian



	SPRINKLER	
RAIN	T	F
F	0.4	0.6
T	0.01	0.99

RAIN	
T	F
0.2	0.8

		GRASS WET	
SPRINKLER	RAIN	T	F
F	F	0.0	1.0
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.99	0.01

<u>**Bayes' Theorem**</u> allows to combine the conditional probabilities to propagate the **evidence** along the network **in the direction of interest.**

$$(\ \vee\ ) = \frac{(\ ,\ )}{(\ )}$$

i.e. "What is the probability that it is raining, given the grass is wet?"

Bottom-up

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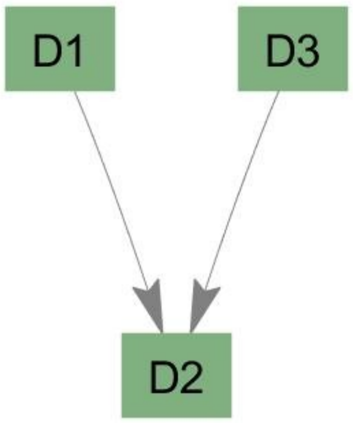
Bayesian Networks
How does it work

D1=d_1	D1=$\underline{d}_1$
0.8	0.2



D3=d_3	D3=$\underline{d}_3$
0.75	0.25

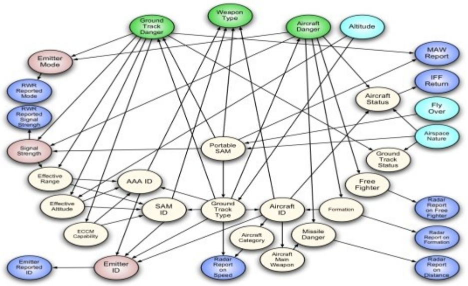
		D2= d_2	D2= $\underline{d}_2$
D1=d_1	D3=d_3	0.1	0.9
D1=$\underline{d}_1$	D3=d_3	0.4	0.6
D1=d_1	D3=$\underline{d}_3$	0.14	0.86
D1=$\underline{d}_1$	D3=$\underline{d}_1$	0.7	0.3

$$(\quad 1,\quad 2,\quad 3)=P(D2|D1,D3)\,P(D1)\,P(D3)$$

Bayesian Networks
Scope and strategies

Capture
Complexity

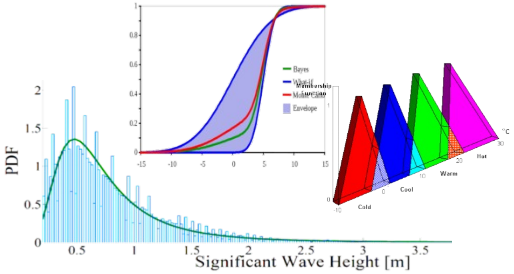
Bayesian Networks



+

Represent
Uncertainty

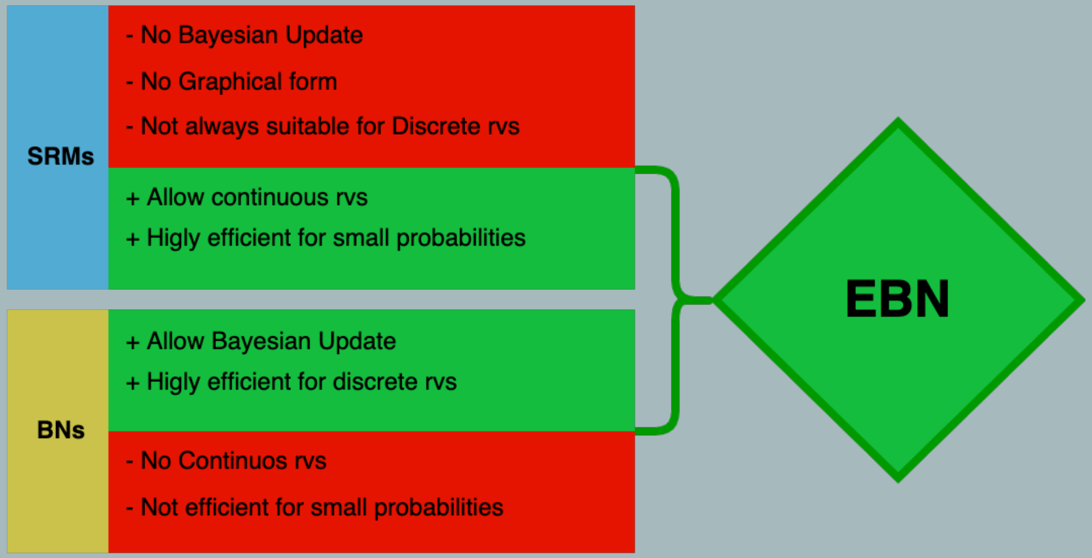
Suitable models



Enhanced Bayesian Networks

EBN properties

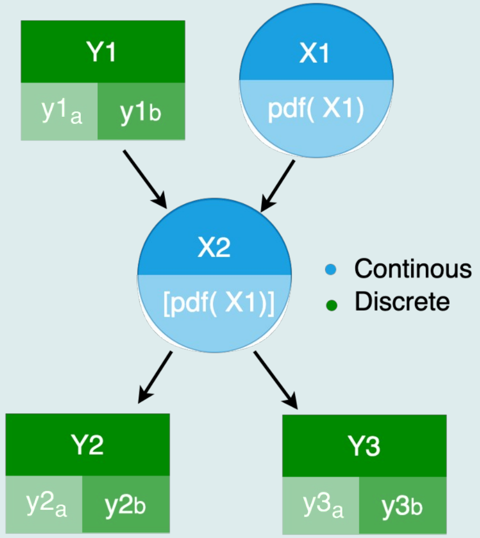
IDEA



eBNs (*BNs Enhanced with SRM*) are a tool able to:

- Implement Discrete and Continuous rvs
- With arbitrary distributions
- And any dependency

HOW



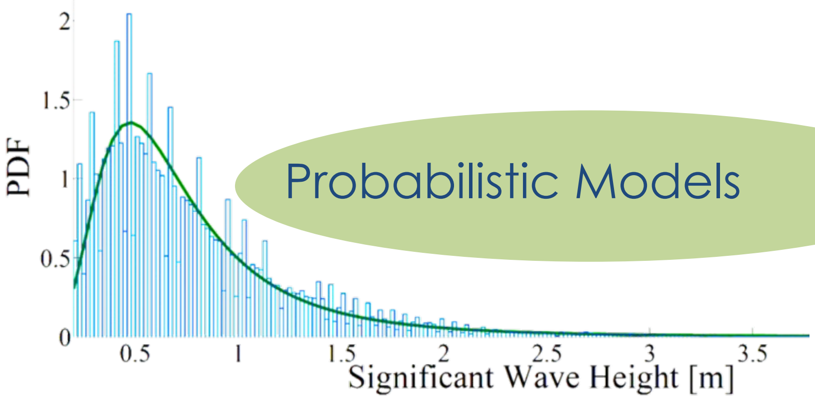
Formally

- **Discrete nodes** have a *finite sample space*
- **Continuous nodes** are *vectors of continuous rvs*
- **System pdf** is expressed by the combined effect of continous and discrete rvs

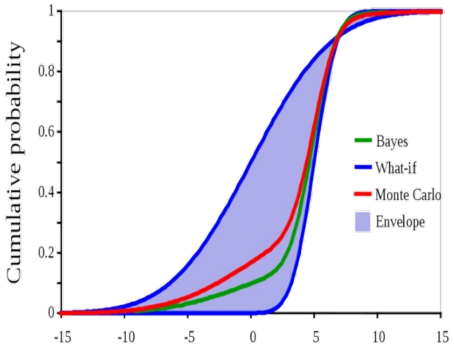
System pdf: $(\;) = \prod_{\in} (\;|\;[\;]) + \prod_{\in} (\;|\;[\;])$

The problem of the evaluation of discrete probabilities (or pdf) of each node with at least one continuous parent has the same mathematical form of a System Reliability Problem!

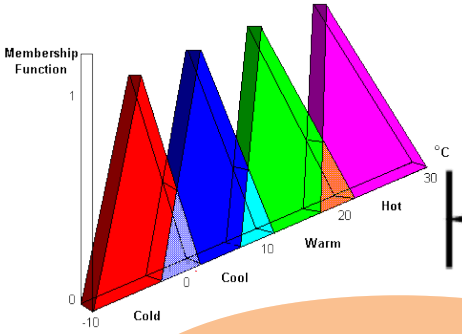
Enhanced Bayesian Networks
Different form of information

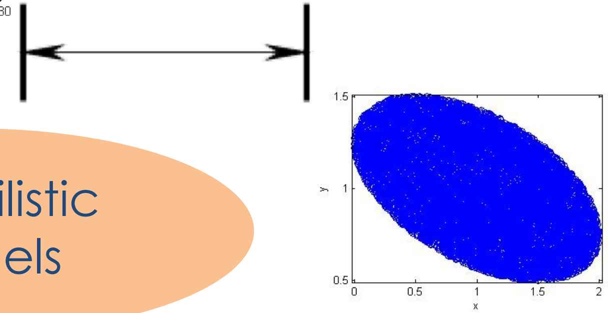


Probabilistic Models



Imprecise Probability Models





Possibilistic Models

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Numerical Implementation of EBNs

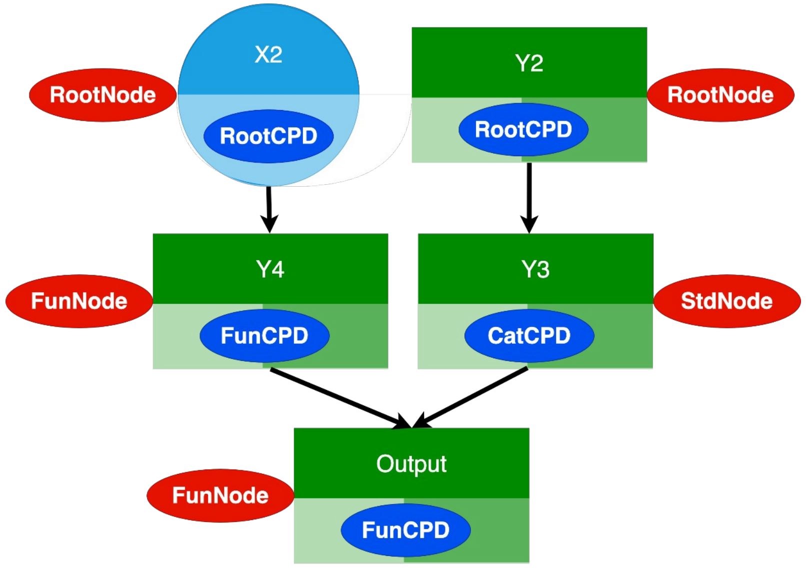
The implementation of the EBN is the main ongoing task underlying <u>all future research</u>!

- Developing a **general purposes library for exploiting eBN in any application**
  - Under development in Julia
  - Based on UQ.jl Library (developed @IRZ, covers the Structural Reliability Methodologies and uncertainty quantification)
- Current State of the eBN framework
  - able to deal with continuous and discrete events (nodes)
  - able to deal with ExternalModel as function that connects one event with its parents
  - able to use different advanced Monte Carlo simulation to get the probability of failure
  - Not able yet to consider evidences on continuous events (WIP)
- Why Julia?
  - More feature complete programming language
  - Free to use and distribute
  - Faster than MATLAB and easily parallelizable (both code and license…)

Numerical Implementation of EBNs

Implemented so far

- **CPDs**
  - Structure to define the Conditional Probability Distribution (Discrete or Continuous) as:
    1) RootCPD
    2) StdCPD
    3) FunctionalCPD
- **Nodes**
  - Structure to define the nodes of the eBN (Discrete or Continuous) as:
    1) RootNODE
    2) StdNODE
    3) FunctionalNODE (both Function or ExternalModel)



BayesianNetworks

- Structure to define the the Bayesian Network as Direct Acyclic Graph and perform the evaluation of each JointCPD given any evidence, as:
  1) StdBayesianNetwork
  2) EnhancedBayesianNetwork

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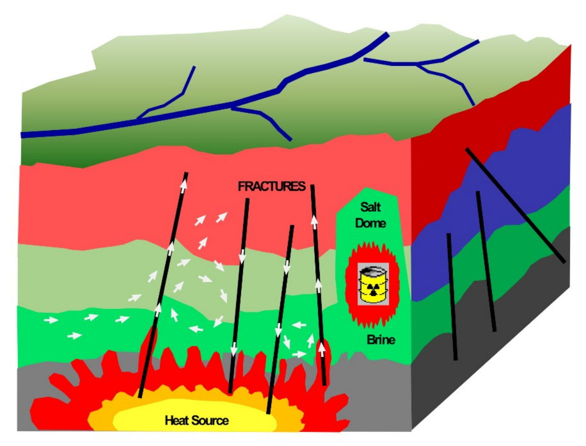
Thermo Hydraulic Model – Problem definition

Case of Study	Risk Assessment

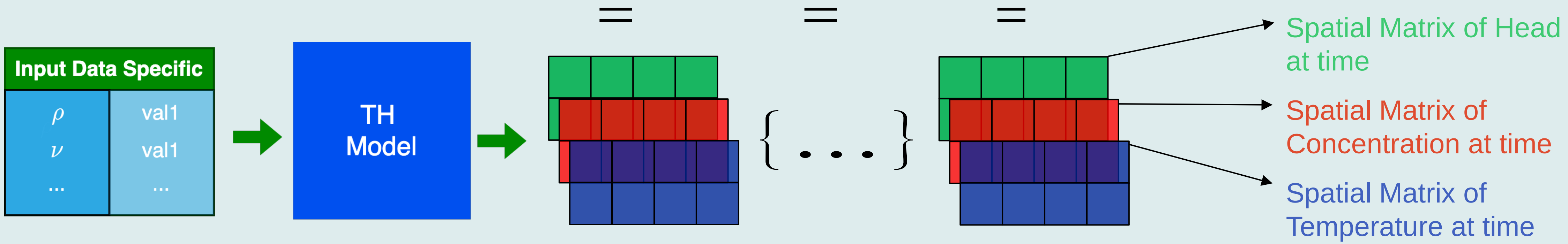
- **The Salt Dome Problem**:
  - Transport of solute (radioactive contaminant) due to groundwater flow within a salt dome (salt dissolution affects flow velocity and vice-versa)
- FE model (smoker.exe) is used to obtain matrices of head, temperature and concentration values at different time.



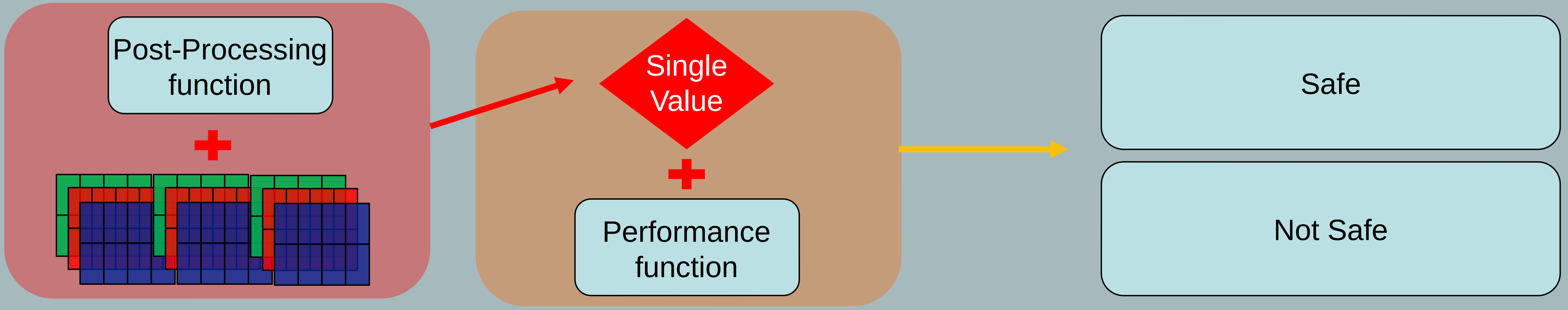
- Identification of the scenarios the affects FE model's inputs, and evaluation of the consequent outputs to determine if the final state is safe or not
- In order to distinguish between safe/not-safe salt dome's final state the FE output (matrices) needs to be
  - Post-Processesed to obtain single values
  - Evaluated through a Performance Function to obtain a boolean output

Thermo Hydraulic Model – Quantities of interest

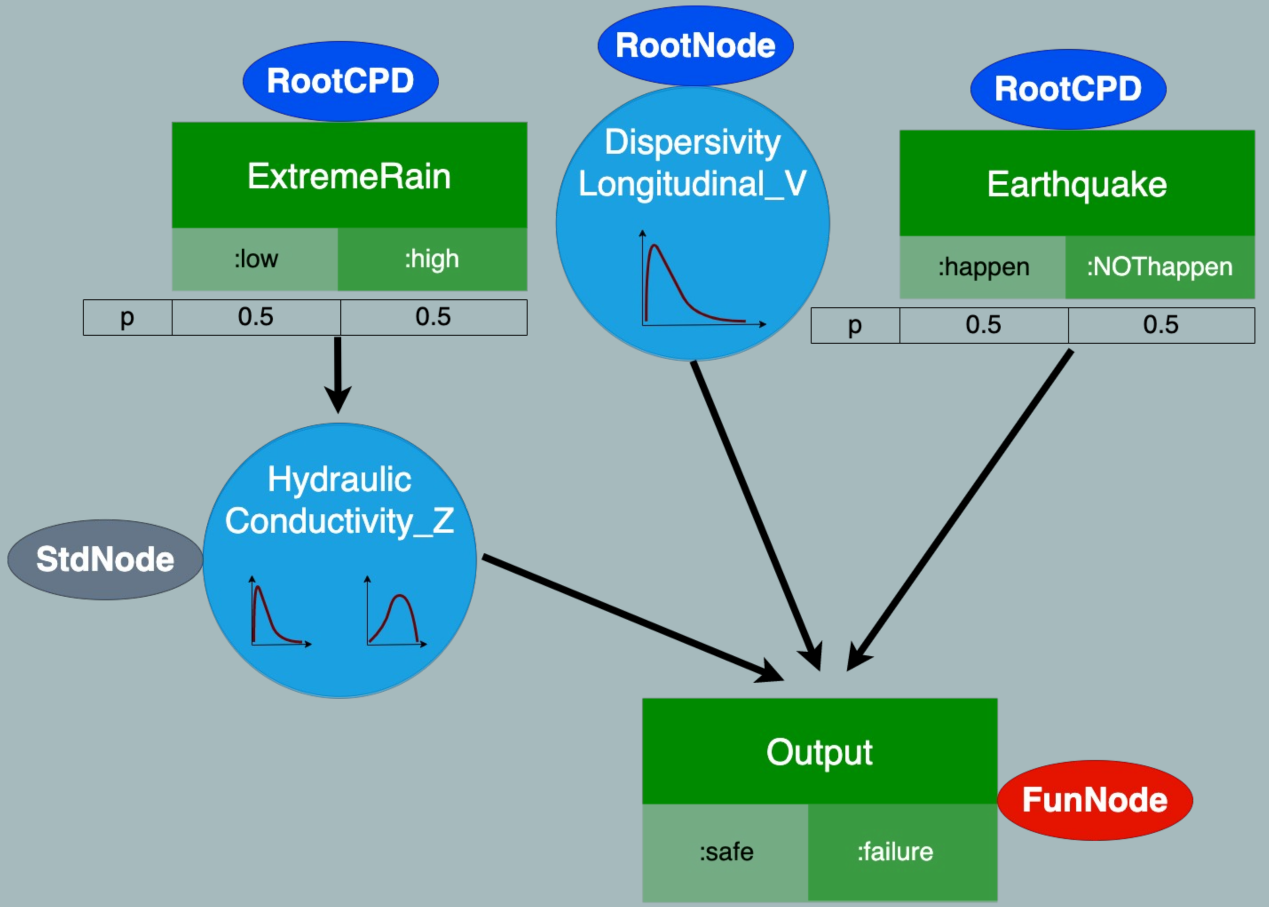
Model’s output



Post-Processing + Performance Function



Thermo Hydraulic Model – Proof of Concept of EBN



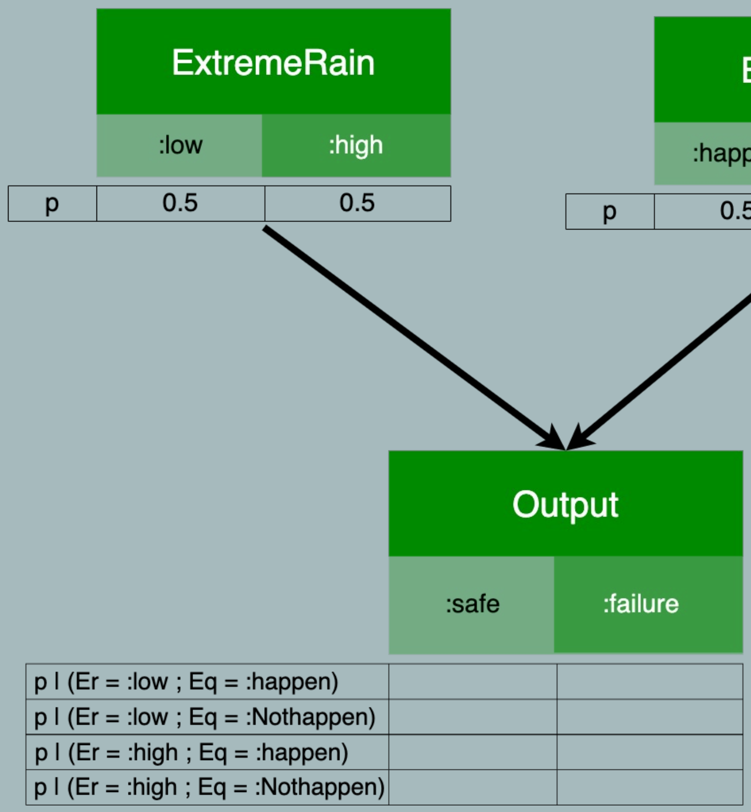
Proof of Concept

- Nodes:
  - 2 Discrete Root Nodes
  - 1 Continuous Root Node with distribution given by a Truncated Normal Distribution.
  - 1 Continuous Standard Node with distribution dependent on its parent
  - 1 Discrete Functional Node where the CPD is defined through the TH_model

- Model:
  - The TH_model used to obtain the probabilities of failure is a simple one with 1 specie, no heat transfer considered and a coarse mesh.

Thermo Hydraulic Model – Reduced Bayesian Network

Reduced Bayesian Network

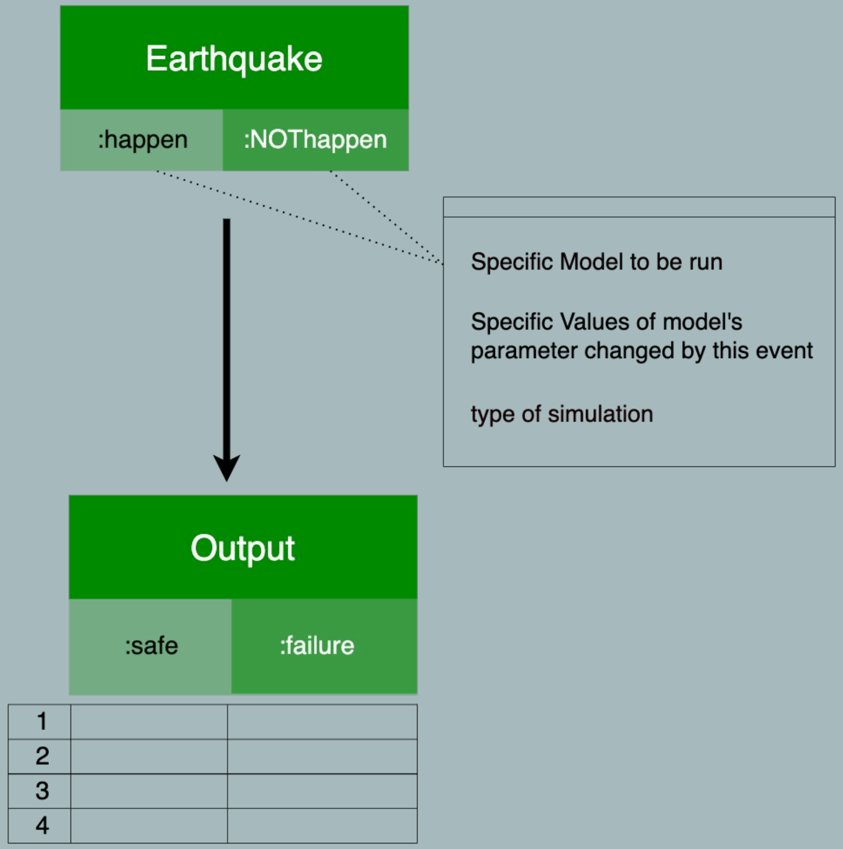


- Reduced Bayesian Network (rBN)
  - Is the network obtained after applying the “Node Elimination Algorithm”
  - rBN defines the number of Structural Reliability Problem that have to be solved in order to obtain the probabilities of failure

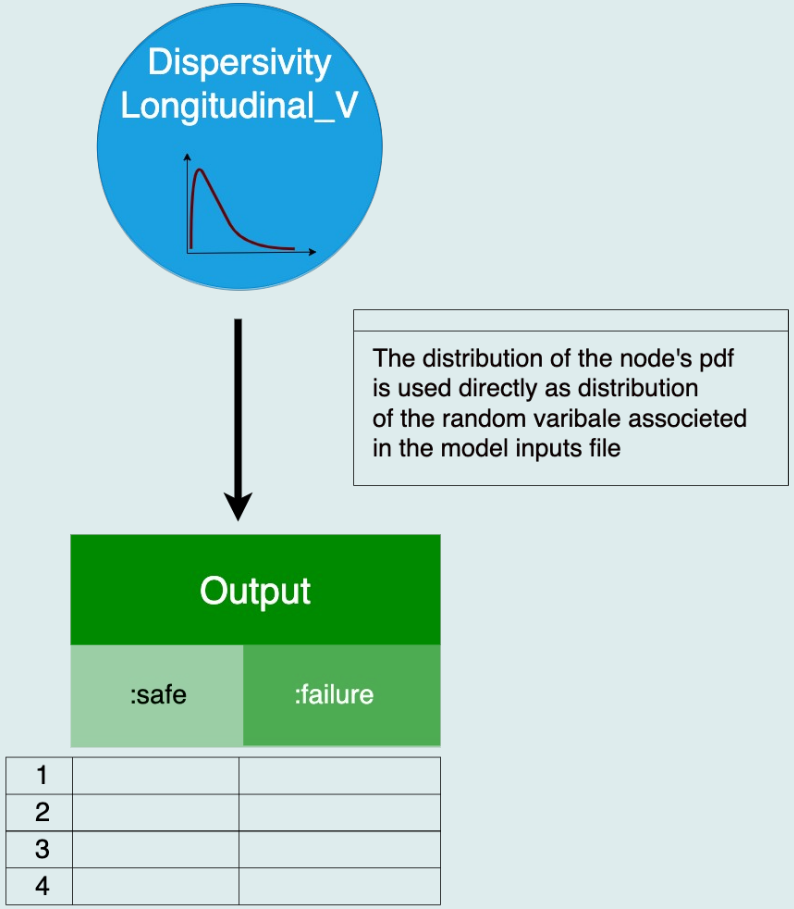
- Structural Reliability Problems (SRPs)
  - 4 SRPs needs to be solved, one SRP for each combination of the parents determination in the rBN

Thermo Hydraulic Model – Nodes and External Models

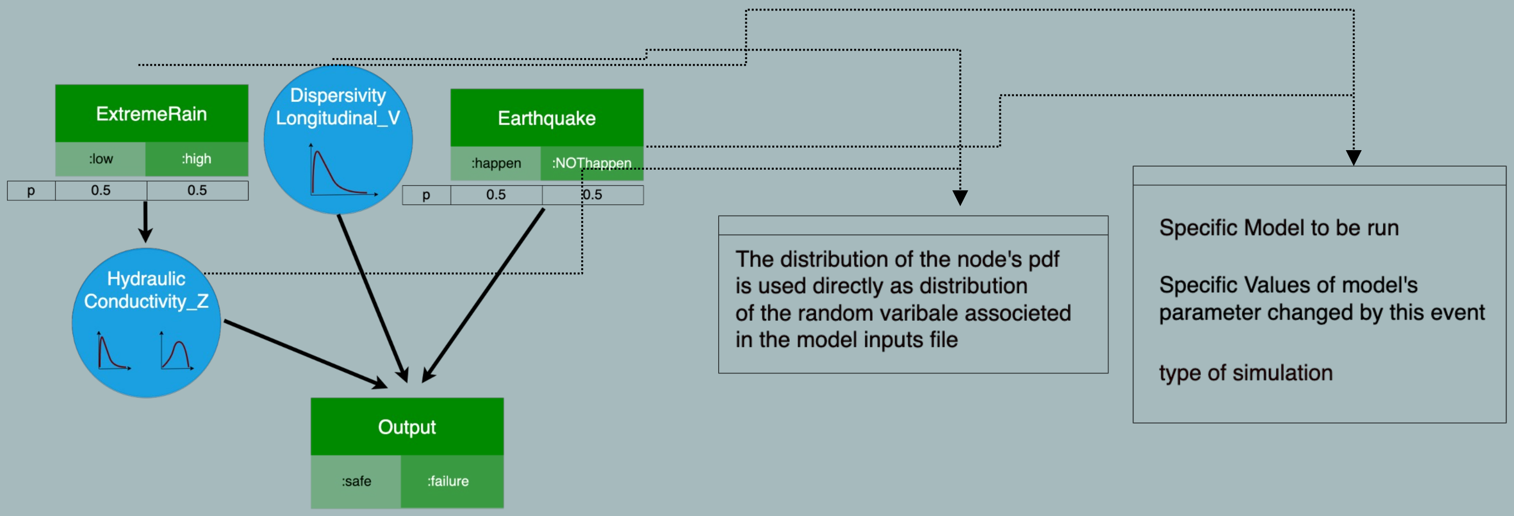
Discrete Parents Node



Continous Parents Node



Thermo Hydraulic Model – Results



PERFORMANCE FUNCTION

Max{Concentration} < 2.2

	Output	
	:safe	:failure
:low ; :happen	1-x	0.02250
:low ; :Nothappen	1-x	0.00125
:high ; :happen	1-x	0.02875
:high ; :Nothappen	1-x	0.00750

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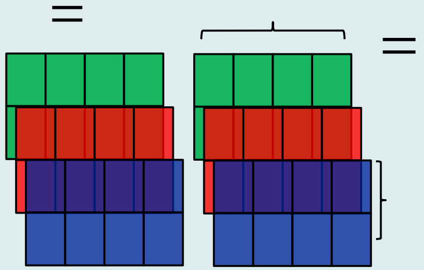
Surrogate Model - ANN

FE Models are too computational expensive in a framework where are required to be run several times in different scenarios, especially when low probability of failure have to be established

TH Model *not simplified version*

$\sim 24h \quad \forall$ simulation



$$Dim = Par + RVs$$

$$Dim = t_{disc} * x_{disc} * z_{disc} * outputs$$

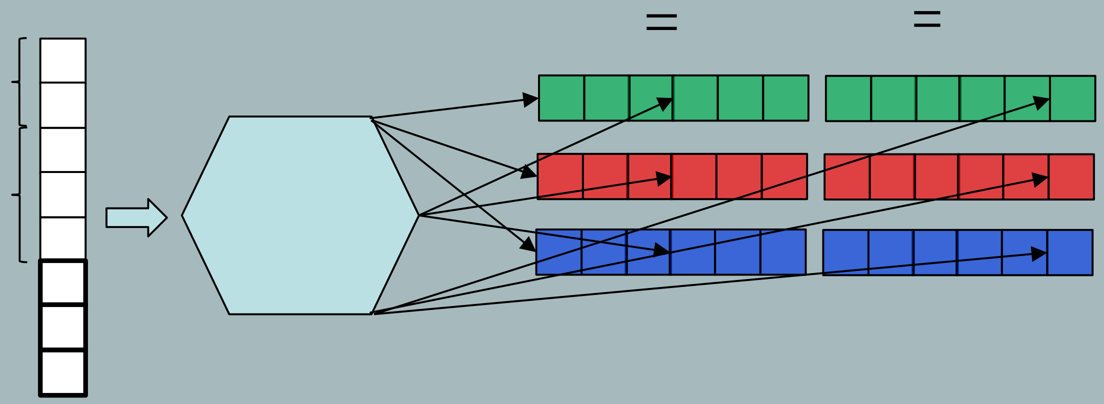
Output

	1 or 2
	10ˆ2
	10ˆ2
outputs	3 [c;T;h]

With a 24h simulation we obtain 1 output sample of 10ˆ4 dimension!

Flattening

- **Reduce Output Dimensionality**
- **Increase Dataset size**



Instead of predicting matrices, **the ANN will predict a triplet (c;T;h) for a specific time-spatial coordinate (x;z;t)**

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Surrogate Model - PINN

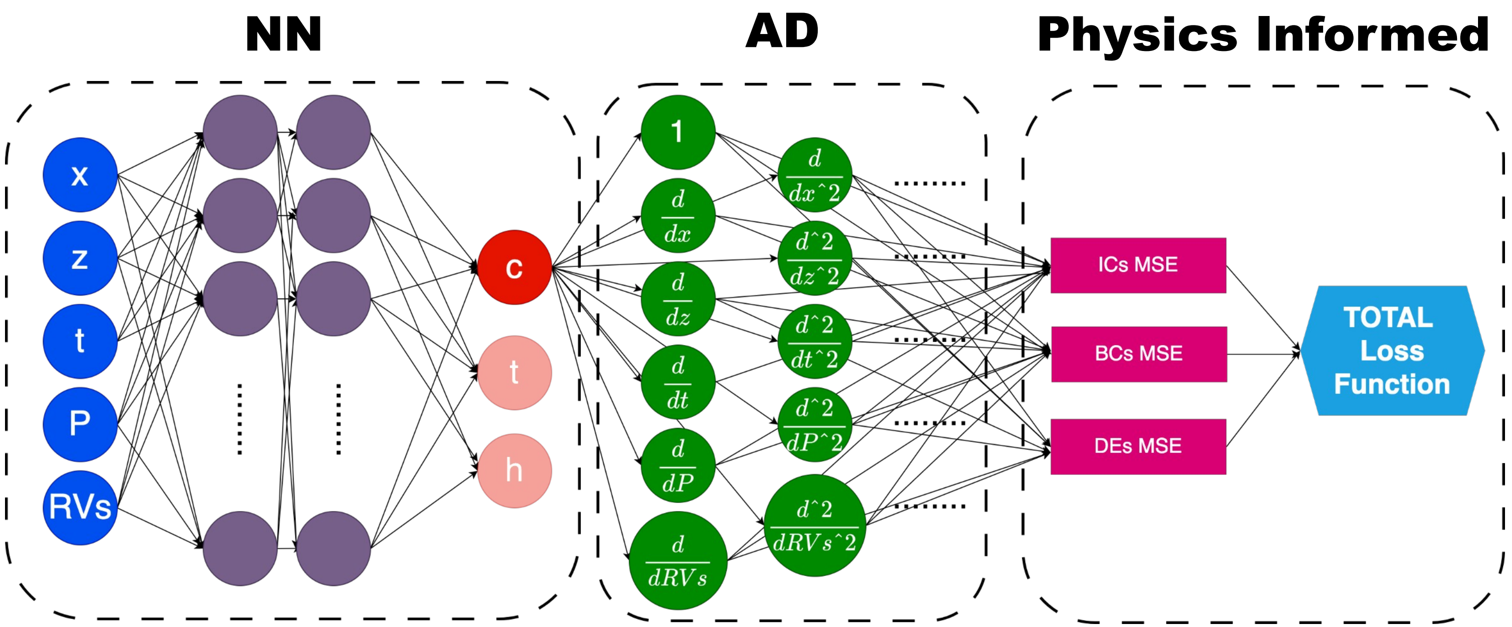
WHY?

- ANN should approximate a Model, not real world
- In ANN framework, computation of derivatives (of any order) with respect to any input/s is computationally cheap

IDEA

“Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations” M. Raissi

- **Including Prior Scientific Knowledge in the ML workflow into the NN loss function.**
- The most important contribution to NN loss function is taken as the residual of DE



CHALLENGE

Establish the differential equations, boundary and initial conditions that address the physical part of the PINN, and implement them in order to obtain the residuals!

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Next Step

eBN Library	RADON Project
• Finalise ‘**EnhancedBayesianNetwork**’ structures adding the capability to deal with evidences on continuous nodes • Identification and implementation of **test cases** (e.g. *Straub 2019 – Bayesian Network Enhanced with Structural Reliability Methods: Methodology*)	• Identification of the events (eBN nodes) and their influences on THC model’s inputs (e.g. *NEA report - Updating the NEA International FEP List An Integration Group for the Safety Case (IGSC) Technical Note*) • Implementation of Salt Dome case • Developing Surrogate Model for TH Model (ANN or PCE or GP) • Present extended abstract at ESREL 2023

Upgrade eBN to imprecise probabilities

- Introduction of ‘Imprecise Probability’ through Interval Variables => Enhanced ‘Credal Networks’
- Efficient simulation tools to avoid “double loop” computational expenses